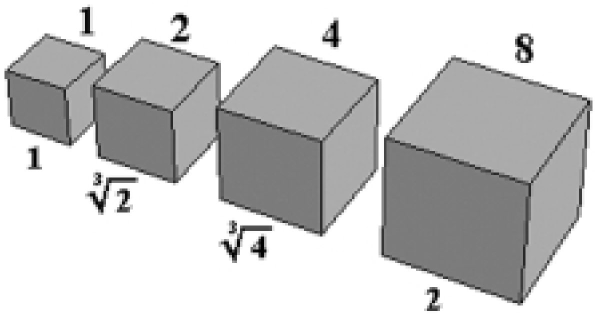
Box 13

How Cubic Roots Are Defined Algebraically

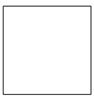
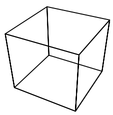
From the Greek studies of the line, square, and cube came an understanding of simply, doubly, and triply extended self-similar action. For example, the triply extended action of a cube necessitates two means between the extremes. This gives an idea of cubic roots (**Figure 1**).

FIGURE 1



It is easy enough for us to retrospectively apply the symbols x, x^2, x^3 to lines, squares, and cubes, respectively. But to what geometry do x^4, x^5, etc., correspond? (**Figure 2**)

FIGURE 2

x	x^2	x^3	x^4
			?

One solution to this paradox (preferred by petulantly childish formal mathematicians) is shown in **Figure 3**:

FIGURE 3

x	x^2	x^3	x^4

Ah, what a relief—with that pesky geometry out of the way, we can enjoy the unfettered freedom of manipulating symbols with assumed self-evident properties! We can simply recognize that x^3 means x times x times x; no troubles here! We can add and subtract too! $5-3 = 2$. And if we want $2-6$, we'd get -4. Hmm, that's a new type of number I did not mean to make with my self-evident numbers, but what of it?

Continuing, we can make equations: like $x^2 = 4$, which we can solve with $x = 2$, and also our "negative" number $x = -2$. We could even say $x^2+4 = 0$, which has as its answer. . . . Well, let's see. . . . Using the rules of algebra, $x^2 = -4$, but what on earth squared is -4? Both 2^2 and $(-2)^2$ are $+4$, not -4. Well, even if it makes no sense, we can use our rule to take the square root of both sides and get $x = \sqrt{-4}$. Now, this corresponds to no real magnitude, but, who cares? Let's use it anyway!

In fact, looking at $x^3 = 8$, we get no less than three solutions, only one of which even makes sense: 2, $-1+\sqrt{-3}$, and $-1-\sqrt{-3}$! Where are these strange numbers coming from? What is the source of these foreign intrusions into *my* view of the universe? Don't I have the personal right to look at things from my own point of view?

—*Jason Ross*