Box 4

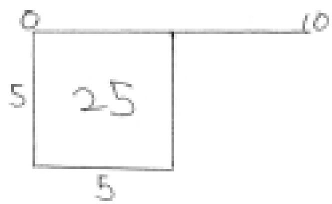
Cardan and Complex Roots

Archytus performed a Promethean act, when he discovered a *Sphaerics*-guided solution to the life-and-death paradox of doubling the cube. For Archytus, that solution lies not in the visible domain of the cube itself, but belongs to a higher domain, where human creativity dances with universal principles, what Gauss has since called the complex domain. From that time to the present, repeated acts of contempt have been perpetrated against Archytus, by those heirs of the legacy of Aristotle and Euclid, who, on behalf of their oligarchical masters, wish to rob man of his fire, and replace it with soulless analytic formulas.

It was more than 1,100 years after Diophantes, the Greek father of algebra, who had developed his mathematics in the dwindling tradition of the Pythagoreans, that Gerolamo Cardan first introduced (in approaching the problem of squaring and cubing) the idea of complex roots, as formal solutions to algebraic problems. For example, if given the equation $x^2-10x+40$, the laws of algebra state that for an equation with rational coefficients, the first coefficient (i.e., 10) will be the sum of the solutions, and the last term (i.e., 40) will be the product of those solutions.

For the notorious gambler Cardan, acting in the empirical tradition of Al-Khowarizmi (famed for the notion of completing the square), this becomes a

FIGURE 1



problem of finding a way to divide a line of 10 units, in such a way, that the two parts multiplied will equal 40 (**Figure 1**).

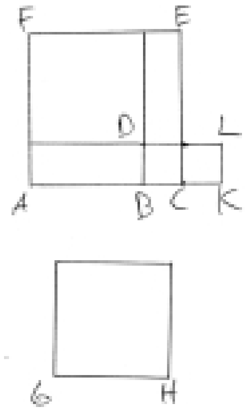
But since the greatest area that can be created through this process (a square) has an area of 25, the problem is considered physically absurd, but algebraically solvable, if we allow for numbers of the form $(a+b\sqrt{-1})$; in this case, $(5+15\sqrt{-1})$ and $(5-15\sqrt{-1})$. Quantities of this type became know as imaginaries, and they haunted Cardan as he tackled the physical problem of cubing. Unlike Archytas, who asked which complex *action* has the power to produce cubic magnitudes, Cardan started, not with action, but with the sense-certain nature of material cubes and their algebraic derivative.

He laid out his cubic problem thus: "For example, let the cube of *GH* and six times the side *GH* be equal to 20. I take 2 cubes *AE* and *CL* whose difference shall be 20, so that the side *AC* by side *CK* shall be 2 . . . (**Figure 2**)."

From here Cardan's equation for general solutions to cubic problems "falls out" algebraically.

Apply to the equation $x^3-12x=10$,

FIGURE 2



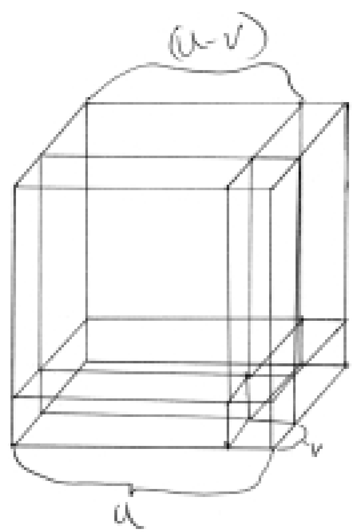
the method prescribed by Cardan, which is in fact purely analytical, despite his request for an initial diagramming of a cube (**Figure 3**):

We let $u^3-v^3 = 10$ and $u^3 \times v^3 = -64$, and consequently $u \times v = -4$.

If now we put in $u-v$ for x, we have:

$$(u-v)^3-12(u-v) = u^3-v^3,$$
$$u^3-3u^2v+3uv^2-v^3-12u+12v = u^3-v^3,$$

FIGURE 3



$3uv(v-u) = 12(u-v)$.

And since $uv = -4$, then $12(u-v) = 12(u-v)$.

And therefore, $x = u-v$ is in accord with our original premises.

And since $u^3 = 10-v^3 = 10+64/u^3$, and because $u^3v^3 = -64$, we then have $u^6 = 10u^3+64$: a quadratic, that can be solved using the age-old quadratic formula: $-b/2a \pm \sqrt{(b^2-4ac)}/2a$ (a formula easily derived from Al-Khowarizmi's work on completing the square).

Using that formula, we come to the "imaginary" solutions:

$u = 5 \pm (\sqrt{-156})/2$,

$v = -4/[5 \pm (\sqrt{-156})/2]$,

$x = u-v$

$= 5 \pm (\sqrt{156})/2 + 4/[5 \pm (\sqrt{-156})/2]$.

Again, the algebra, applied to what is in actuality a physical problem, has produced something ambiguous and unknowable.

When carrying out algebraic investigations of literal squares and cubes, the occurrence of complex quantities, as solutions, is a total paradox. For what is a negative cube in the material world? (Is $\sqrt[3]{-x^3}$ the edge of a cube whose volume is $-x$?) And, even more absurd, what would something like x^4 or x^5, etc., "look like"? Thus, geometry, when condemned to "flat Earth" three-dimensional Euclidean space, loses the name of action, taking on the character of a stiffened corpse, no longer susceptible to cognitive interaction; and algebra becomes a pseudo-science, practiced to maintain an "ivory tower" fantasy.

The Gambler de Moivre

It was continuing in this depraved tradition, that a close ally and co-conspirator of Sir Isaac Newton, Abraham de Moivre (whose chief form of employment was as an advisor to the gamblers of his day, much like the bulk of today's mathematicians who work for the various casino-like hedge funds of Wall Street) seems to be the first to have found it convenient to apply trigonometric laws (although with no connection to the circular action from which those laws were born), to his sadistic investigation of the cubic roots. In one particular stab, he begins with what he calls an "impossible binomial" $(a+\sqrt{-b})$, and seeks to find its cubic roots. Knowing, from his intense indoctrination in mathematical textbooks, that the trigonometric equation $4\cos^3 A/3 - 3\cos A/3 = \cos A$, associated with the trisection of an angle, could be made to yield 3 solutions, he set out to contort the algebraic equation, for a cubed binomial $(x+\sqrt{-y})^3 = x^3+3x^2\sqrt{-y}-3xy-y\sqrt{-y}$ into a form which is algebraically akin to that of the trigonometric formula. (That is, $4x^3-3mx = a = 4(x/r)^3-3(x/r) = c/r = 4x^3-3r^2x = r^2c$).

Once that's been achieved, de Moivre carries out a series of algebraic manipulations of the trigonometric equation, winds up with three angular solutions, "applies the table of sines," and gets three new fractions, which he then plugs back into his previously derived algebraic equation, fondles it a bit, and ends up with the three desired algebraic solutions, two of which are "imaginary" $(a+\sqrt{-b})$.

So, like Cardan, he winds up with algebraic magnitudes, that if squared, would be said to have produced a negative area—a paradox, and doubly so in this case, in that this was achieved by using circular (trigonometric) functions. But, for de Moivre, whose creativity was crippled by that "drill and grill" abuse at the hands of his "ivory tower" controllers, there is no paradox. The fact that his algebraic investigations lead him to the use of circular functions, where $z = x+iy$ becomes $z = r(\cos\varphi + i\sin\varphi)$, and finding the cube root takes the form of finding the cubed root of a radius ($\sqrt[3]{r}$) and trisecting the angle ($\varphi/3$), is only formally consequential and ontologically unknowable. For de Moivre there is no action, or higher ordering principles at work, only the "imaginary" shadow world idea of algebra and its "right answers."

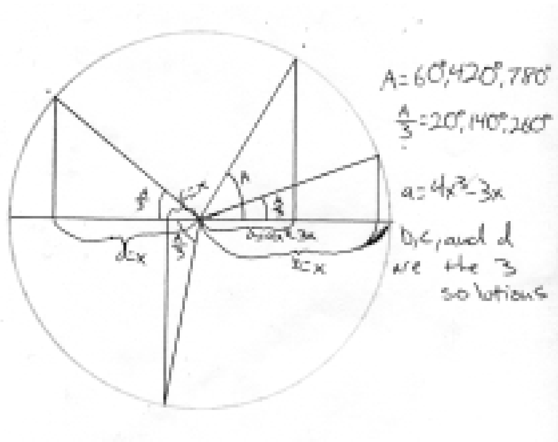
Unfortunately, due to his obsession with, or better, possession by formal algebra, and his absolute denial of the knowability of the principles of action, characteristic of constructive geometry, the paradoxical occurrence of complex roots, and the handling of them by trigonometric properties, never provoked de Moivre to ask those questions of cause, which spawned the hypothesis made by Gauss, that the "imaginaries" were reflections of an action, which is ontologically transcendental.

It was his mind's shackling at the hands of algebraic formalism, which barred him from looking to the physical geometry behind the shadows of his formulas, to discover, that what he had deemed to be "impossible," were in fact the effects of a true physical action. For example, in the physical construction for the trisection of the angle, two of the solutions that would have appeared to de Moivre to be imaginary, are in fact real (**Figure 4**).

In other words complex numbers are not arithmetic quantities, but rather

Box 4 continues on next page

FIGURE 4

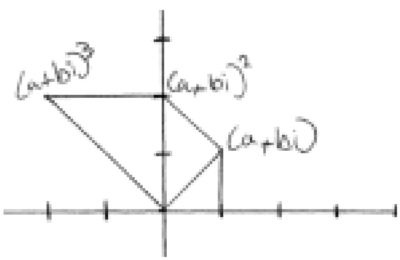


Three solutions to cubic function in the complex domain: Tripling the angle of any of the three solutions of 20°, 140°, and 260° will bring you to the desired 60°.

haunts, of a knowable, higher action, which subsumes the algebra. So it was Gauss, who was left to re-stoke that flame of Pythagorean *Sphaerics,* which had been reduced to smoldering ashes by those followers of the cult of Newton (**Figure 5**).

It was one of de Moivre's students, d'Alembert, who thought he could totally purge science of geometry, by seemingly introducing it in his attempt at a proof of the fundamental theorem

FIGURE 5



Cubing a complex magnitude ($a + b\sqrt{-1}$ in the complex domain, a combination of rotation and extension.

of algebra. In effect, he employs what is commonly known today as the "plug and chug" method of Cartesian point-plotting, of trying to close in, getting infinitely closer to the solution.

So, given the algebraic problem of $x^2 + 1 = 0$, the method of d'Alembert calls for simply plugging all the possible real values in for the variable and plotting the variable as the ordinate and the function as the abscissa (**Figure 6**).

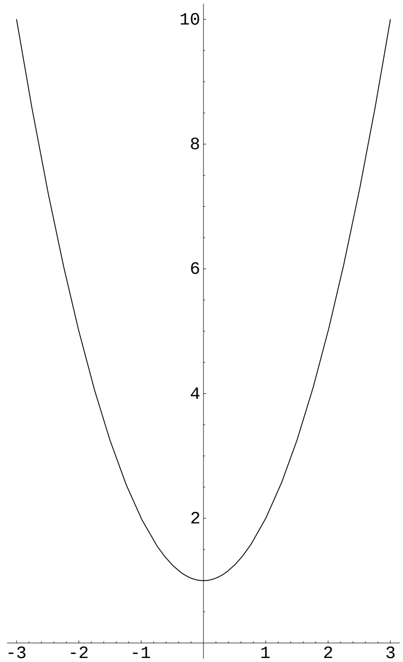
For cases where the reals don't lead to an answer, such as the $x^2 + 1 = 0$ problem, d'Alembert calls upon the magic of the imaginaries, and says we can use quantities of the form $a + b\sqrt{-1}$ to yield solutions. If we plug in all the possible $a + b\sqrt{-1}$ quantities, we produce a curve that does cross the imaginary ordinate, giving us our two answers (**Figure 7**).

Gauss's Critique

To this, Gauss says of d'Alembert's proof: "It is proper to observe, that d'Alembert applied geometric considerations in the exposition of his proof and looked upon X as the abscissa, and x as the ordinate of a curve . . . but all his reasoning, if one considers only what is essential, *rests not on geometric but on purely analytic principles,* and an imaginary curve and imaginary ordinate are rather hard concepts and may offend a reader of our time."

This is the crux of Gauss's attack on the whole of the works of Euler,

FIGURE 6



Equation $X = x^2 + 1$:

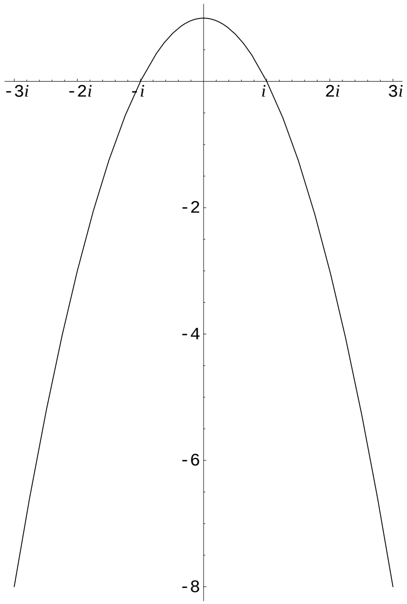
x	-3	-2	-1	0	1	2	3
X	10	5	2	1	2	5	10

d'Alembert, et al., in his 1799 proof of the Fundamental Theorem of Algebra: Their proofs were conspicuously void of constructive geometry, and hence human creativity. At best, they simply investigated that which is, as opposed to asking the question: What has the power to make possible that which is?

It is no hyperbole to say that this fight, over the challenge of discovering a solution to the paradox associated with the doubling of the cube, is a life-and-death one.

As history has shown, and as LaRouche's discovery has made known, man only survives when he

FIGURE 7



Equation $X = x^2 + 1$:

x	$-3i$	$-2i$	$-i$	0	i	$2i$	$3i$
X	-8	-3	0	1	0	-3	-8

progresses, and he only progresses when he applies his uniquely human power of cognition to those paradoxes which the universe communicates to us. Constructive geometry, in the complex domain, of the tradition of Archytus, through Gauss and Riemann, is the embodiment of those creative acts, which not only express, but also strengthen, that relationship between man and the universe. Any attempt to formalize and to degrade such universal problems of physical geometry to the level of the analytic, is nothing short of a crime against humanity, performed on behalf of those whom Dick Cheney calls master.

—*Cody Jones and Chase Jordan*