II. The Unity of Science and Art

POET-MATHEMATICIAN SERIES, PART IV

Sophie Germain

by David Shavin

Feb. 9—China has put on the table the beautiful—and very "American"—mission of wiping out poverty in China by the year 2020. The type of thinking required today to finally wipe out poverty, disease and hunger will involve a level of creativity once described by Lyndon LaRouche as being able to "play ping-pong with the stars." The beautiful composition of a new alliance of nations—pushing the frontiers of plasma physics, fusion technologies, and materials processing, as the economic surplus is deployed to craft massive infrastructure projects throughout the developing world—requires a level of thinking and emotional development that will make future generations stand in awe. This type of thinking is that of the "poet-mathematician." It was also expressed in Plato's Republic *as that of the "philosopher-king"—the almost impossible, but completely necessary development of leaders, who pursue the most difficult paradoxes in astronomy and music, so as to harmonize their souls with the complexities of the development of human communities. After the American Revolution, a youthful genius, Karl Gauss, in what was apparently an obscure mathematical text, went boldly where most others feared to tread. An identifiable, small core of youth, took up Gauss's challenge while he was alive. They were Sophie Germain, Lejeune Dirichlet, Niels Abel, Evariste Galois and Bernhard Riemann.*

Lawfully, and somewhat ironically, the individuals who most seriously, most passionately, took up this mission, have proven to be, as rather unique individuals, the most fascinating exemplars, in their own personalities, of the higher-ordered mathematics. The historically-specific realities of their lives rise to a level beyond mere biographical sidenotes—a level helpful in delineating how they were able to develop such a rigorous and higher-ordered language, appropriate for mapping how the mind intervenes upon the outside world.

Sophie Germain

The case of Sophie Germain closes this series on Gauss's five prime students, all poet-mathematicians.[1]

Though not well-appreciated, Sophie Germain was the first serious student of Gauss's *Disquisitiones Arithmeticae*, or *DA*. Even less appreciated is that her work with Gauss served her as the uniquely appropriate "aesthetic education" for her probe of unseen harmonies of music. Again, as with Dirichlet, Abel, Galois and Riemann, we shall find a non-'mathematical', musical core that guided her work in both science and art. And, again, the historically-specific moral core of Sophie Germain's too-lonely battle

1. The case for Dirichlet was made here: http://www.larouchepub.com/other/2010/3723rebecca_dirichelet.html The other three are listed as Parts I to III, and found at: I. Abel. II. Galois. III. Riemann.

for beauty and truth is identifiable, and it was the driving force of her accomplishments.

Sophie Germain recognized the need for the poet-mathematician. She wrote that decent leaders in normal times may be clever enough. "In times of crisis, however, it's something else. Circumstances become pressing; we must know how to make prompt decisions; we also often need courage, and courage is not necessarily a quality of the clever man." Germain will make the case for the unity of courage and genius, that it requires the mastering of the modalities of Karl Friedrich Gauss—and, perhaps surprisingly, the modalities of J.S. Bach.

Carl Gauss

I. Archimedes and Gauss

In the three years after Gauss's 1801 *DA*, Germain launched into an intensive study of the work. In 1804, she first wrote to Gauss, developing some further implications of his work. She began, "Monsieur—For a long time your *Disquisitiones Arithmeticae* has been an object of my admiration and study." In her letter, she developed a subsection of Gauss's "4n+3" primes, now called "Germain primes," to develop an approach to proving Fermat's last theorem. She signed the letter "Monsieur LeBlanc," as she feared she would not be taken seriously if the name on the letter were that of a woman.[2] Gauss wrote LeBlanc, pleased that he had taken up "the research to which he [Gauss] devoted the most beautiful part of his youth. . ." After the third letter from "LeBlanc," Gauss told Heinrich Olbers, his astronomical collaborator: "I am amazed that M. LeBlanc has completely mastered my Disq. Arith., and has sent me very respectable communications about them."

When in 1807, Gauss discovered that his correspondent was actually a woman, one Sophie Germain, he was more than delighted:

"But how to describe to you my admiration and astonishment at seeing my esteemed correspondent Monsieur Le Blanc metamorphose himself into this illustrious personage who gives such a brilliant example of what I would find it difficult to believe. A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare. One is not astonished at it—the enchanting charms of this sublime science reveal only to those who have the courage to go deeply into it. But when a person of the sex which, according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents and superior genius. Indeed nothing could prove to me in so flattering and less equivocal manner that the attractions of this science, which has enriched my life with so many joys, are not chimerical, as the predilection with which you have honored it."

But even more telling was the story behind the revelation of her identity. During Napoleon's 1806 invasion of Germany, Germain had requested a family friend, General Joseph-Marie Pernety, to intervene, and to extend protection to Gauss. When Gauss was told that his protectress was one Sophie Germain, he was puzzled, saying that the only woman that he was acquainted with in Paris was the wife of an astronomer-friend, and not anyone named "Sophie Germain." The general reported this back to Germain, and she wrote to Gauss, explaining that, in fact, he did know her, that she was his correspondent, M. LeBlanc. That occasioned

2. As part of her disguise, Germain instructed Gauss that he could write back to "LeBlanc" at the address of one Silvestre de Sacy. This was a family friend, Antoine Isaac, Baron Silvestre de Sacy, a linguist from a Jewish family of Paris. Of some note, while Germain was decoding Gauss's *DA*, the linguist was working on the decoding of the famous Rosetta Stone. (Later, de Sacy personally initiated both Champollion and Thomas Young into the project.)

Gauss's letter (above). However, what she didn't explain to Gauss was the psychological horror behind her actions. For her, Gauss represented the precious, rare mind of an Archimedes; and she was horrified that Gauss might receive the same treatment as Archimedes had—murdered by an occupying force.

The key, formative and driving experience for Germain was when, as a thirteen-year-old, the turbulence, confusion and violence of the 1789 revolution sent the sensitive Sophie into her father's library, where she delved into Montaclu's *Histoire des Mathematiques*.[3] There, amidst the stories of scientific investigations and discoveries over thousands of years, Sophie made an intimate friend of the great mind of Archimedes. But she learned, to her horror, that such a treasured man, in the midst of his intellectual concentration, was struck down by a Roman soldier. There, the inspiring and beautiful pursuit of truth was confronted with the brutally senseless. During the senseless horrors of the next five years in Paris, culminating in the infamous "Terror," the sensitive teenager pursued her struggle for eternal verities.

From that point on, Sophie kept her bond with Archimedes. Despite familial and social pressures to adopt a more traditional position for a woman, and with no hope of a professional career, Sophie pursued her mission. Five years later, at eighteen, Sophie took advantage of the availability of lecture notes from the presentations at the newly-founded Ecole Polytechnique. She submitted responses to them under the name of "M. LeBlanc," at that time, the name of an actual student at the Ecole. When the professor, Joseph Lagrange, wanted to meet this Antoine-August LeBlanc, the student with such apt observations, Germain's identity was disclosed. Over the next ten years, various professors would treat the young woman as a talented oddity, the woman-mathematician. Typically, they would offer their own textbooks to her as the proper next step for her self-improvement. Germain was not excited about playing the role of Eliza Doolittle for Professor Higgins, and she avoided such attentions.

Prior to Gauss, it would appear that only Adrien-Marie Legendre took her mind seriously enough to answer questions and engage in dialogue.[4] The revelation in 1807, that Monsieur LeBlanc was actually Sophie Germain, appears to have actually increased Gauss's interest level in his correspondent's character and mental powers. For the first time in three years of correspondence, Gauss described to her three new theorems on cubic and biquadratic residues; however, he deliberately omitted his proofs, as he explained, "…in order not to deprive you of the pleasure of finding them yourself, if you find it worthy of your time… Continue, Mademoiselle, to favor me with your friendship and your correspondence, which are my pride, and be persuaded that I am and will always be with the highest esteem, Your most sincere admirer."

Two months later, Gauss received his three proofs. Germain wrote, "How I have enjoyed reading your three theorems on residues! I have searched for demonstrations of them. I add them to my letter in order to have you judge them.… In attempting to provide proofs for them, I have developed a way of thinking that for me is full of charm."

II. Classical—'A Way of Thinking That for Me Is Full of Charm'

The strategic role of Gauss in this series involves his complete development of Johannes Kepler. Kepler had taken up the challenge in Plato's *Timaeus* and developed the incredible but true, underlying coherence between such "objective" matters as the organization of the solar system and such "subjective" matters as the harmonic ordering of the mind's hearing. The planets were organized just as the human ear heard musical intervals. Now, this is, indeed, what puts the "classical" in classical. The core of the human identity, the mind, is uniquely tuned to be in synch with the most powerful forces in creation, and so to be capable of bringing them under deliberate mastery. A world lacking this characteristic simply would not be classical.

Gauss relentlessly pushed forward this classical double-counterpoint of Kepler. At the same time that Gauss composed his *DA* on the unseen harmonies of the human mind, he also shocked European scientists by showing how Kepler's approach solved the seem-

3. In 1789, her father, Ambroise-François Germain, was elected deputy of the Third Estate for the city of Paris, and was a member of the National Assembly at Versailles. (Apparently, he took public positions against "agiotage," that is, making a business out of currency exchange.) Later, in 1800, he would become a Directeur of the Banque de France.

4. Legendre's 1798 "*Essai sur la théorie des nombres*" impressed Germain. It probably led Germain to Gauss's 1801 work, where she would find both corrections of Legendre and a much fuller development.

ingly unsolvable "objective" problem of the orbit of Ceres.[5] It might be surprising to some, but his *DA* of 1801 is best understood as an intensive exploration and development of the geography of the inner workings of the human mind. How else to understand the miracle of the reciprocity of two different species of prime numbers?

Though Sophie Germain certainly had studied Gauss more than anyone else, she had also read her Kepler. And, as of her completion of the three proofs for Gauss in 1807, she had developed this way of thinking, one that she found "full of charm." It would lead to her winning the scientific prize of the French Academy for her work on the underlying patterns of sound.

5. Jonathan Tennenbaum and Bruce Director, "How Gauss Determined the Orbit of Ceres" https://www.schillerinstitute.org/fid_97-01/982_orbit_ceres.pdf

Quadratic Reciprocity and Political Revolutions

The harmonic patterns in Gauss's residues are no less fascinating than the beautiful Chladni patterns.

Gauss examined how numbers had something in common if they shared the same modulus—that is, if divided by the same number, they yielded the same remainder, or residue. So, 11 and 18 both yield 4 with respect to the modulus 7. Next, Gauss compared the quadratic series—the squares of 1, 2, 3…, that is, 1, 4, 9…—in terms of a given prime-number modulus. Four examples may assist:

Squares	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256
Mod 7	1	4	2	2	4	1	0	1	4	2	2	4	1	0	1	4
Mod 11	1	4	9	5	3	3	5	9	4	1	0	1	4	9	5	3
Mod 13	1	4	9	3	12	10	10	12	3	9	4	1	0	1	4	9
Mod 17	1	4	9	16	8	2	15	13	13	15	2	8	16	9	4	1

Gauss found that every modulus displayed patterns as to how their quadratic residues spread out. First, for a modulus of size n, all the numbers from 1 to n-1, would divide up, with half being residues and the other half, non-residues. Even better, a type of inversion is found: Halfway through the n-1 residues of a modulus of n, the residues would turn around and repeat themselves backwards—or as Bach would say, a *canon al roversio*.

Next, the modulae would divide into two different basic groups of prime numbers. Modulae such as 5, 13 and 17 would fall into one group, called the "4n+1" primes, where each residue had a unique partner, whereby their sum would equal the modulus. (In mod 13, 1 pairs with 12, 4 with 9, 3 with 10.) However, the residue of modulae such as 7, 11 and 19, called the "4n+3" primes, would never have such a residue partner; however, each residue did have such a unique partner amongst the non-residues. (For mod 7 in above example, 6, 3 and 5 are the non-residues that pair, in order, with 1, 4 and 2.) Also, the "4n+1" group always includes n-1 as one of its residues, while the "4n+3" group never does. (For example, mod 13 includes 12 as a residue, while mods 7 and 11 do not include 6 and 10, respectively.) Who knew that prime numbers fell into two such categories? But hold on to your horses.

Gauss proved a fundamental principle of reciprocity amongst the two basic groups of prime modulae, raising inversion to a bold new level. How does a modulus relate to its residue if their roles are reversed? (For example, since 13 is a quadratic residue of mod 23, will 23 be a quadratic residue in mod 13? In this case, 23 in mod 13 is the same as 10, and 10 is a quadratic residue of mod 13.) Inversions can be challenging. A too simple example would be to compare two processes: a) given that one knows who the murderer is, figuring out the steps taken by the murderer, vs. b) not knowing any murderer, coming across a murder scene and coming up with the unique sequence of actions that resulted in all the parts (including the identity of the murderer) being as they are. Deducing a chain of events is not quite the job that its inverse is.

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III. Chladni's Harmonic Patterns and Ben Franklin

The following year, 1808, Paris was seized with the provocative and beautiful displays by Ernst Chladni of the harmonic patterns of plates bowed with a violin bow. Chladni would spread sand upon a surface, so that when the plate was agitated by the stroke of the violin bow on its edge, the sand would congregate upon the nodal lines—thus, displaying the architecture of the dynamics of the plate.[6] The plates—whether of wood, glass or metal—were only a first approximation of the more complex dynamics of an arched (or "vaulted") violin plate.

Chladni described that he had gotten the idea from Georg Lichtenberg, the Leibnizian professor at the University of Göttingen, who had employed various powders, including sulphur filings, to display the patterns of electrical activity on a surface, activity initiated by the discharge of a spark. Lichtenberg, in turn, had been inspired by America's Benjamin Franklin to investigate electrical and magnetic phenomena.[7] Chladni also credits Göttingen's professor of music, J. N. Forkel, for giving him the idea of using a violin bow for his experiments.[8] Forkel also provoked Chladni to attempt a further development of Franklin's

6. Take a moment to examine the formation of the Chladni patterns: https://youtu.be/lRFysSAxWxI

7. Lichtenberg was a student at Göttingen when Benjamin Franklin visited in 1766. Lichtenberg attended the welcome dinner for Franklin, where he heard Abraham Kaestner's keynote address on Franklin's electrical experiments. Later, Lichtenberg would install Franklin's lightning rods at Göttingen.

8. Johann Nikolaus Forkel was an associate of two of Bach's sons. His early, brief biography of Bach has not been surpassed.

Quadratic Reciprocity and Political Revolutions

(Continued from previous page)

Gauss was able to prove the amazing result about this inversion, one called quadratic reciprocity:

1. Amongst the 4n+1 prime modulae, the modulus is a quadratic residue of every one of its own residues, and it is a quadratic non-residue of all of its non-residues. Very symmetric.
2. Amongst the 4n+3 prime modulae, the modulus is a quadratic residue of its 4n+1 quadratic residues, but a non-residue of its 4n+3 residues. Rather anti-symmetric.
3. Further, amongst the 4n+3 prime modulae, the modulus will be a quadratic residue of its non-quadratic, 4n+3 residues; but will not be a quadratic residue of one of its non-quadratic, 4n+1 residues.[1]

Mastering the symmetries and dissymmetries of the harmonic patterns of Gauss's *Disquisitiones Arithmeticae* were at the root of the work of Germain, Dirichlet, Abel, Galois and Riemann—work on the solar system, Chladni plates, Fermat's Last Theorem, the quintic (or the problem of the boundary at the "fifth-power"), shock-waves, general relativity and, yes, good political revolutions.

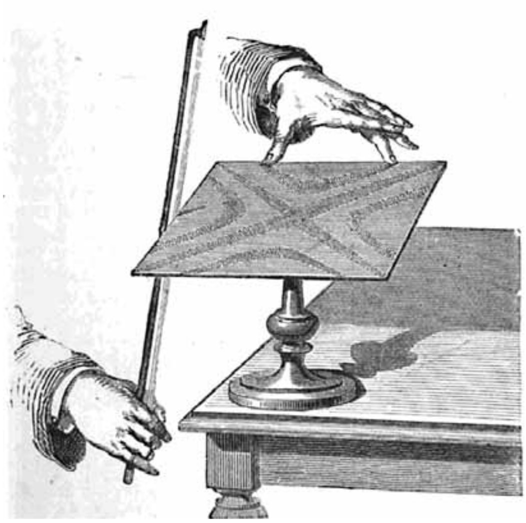
So, perhaps the last word here on Gauss's quadratic reciprocity may be allowed for a good political revolutionary:

All consistent mathematics as such, reflects obviously underlying ontological, axiomatic-like presumptions, which, however strenuously "pure" mathematicians may attempt to hide this fact, are "secretions" rooted in the physical geometry inherent in the processes of the individual human thinking mind. . . [I]t is obvious to me that the real foundation for Gauss's argument for the startling expression of quadratic reciprocity, reflects the implicit reality, that the assumptions of arithmetic are not pure, but, as many of us have insisted, repeatedly over generations, lie within the domain of the ultimately physical geometry of the biology and metabiology of the human mind-function.[2]

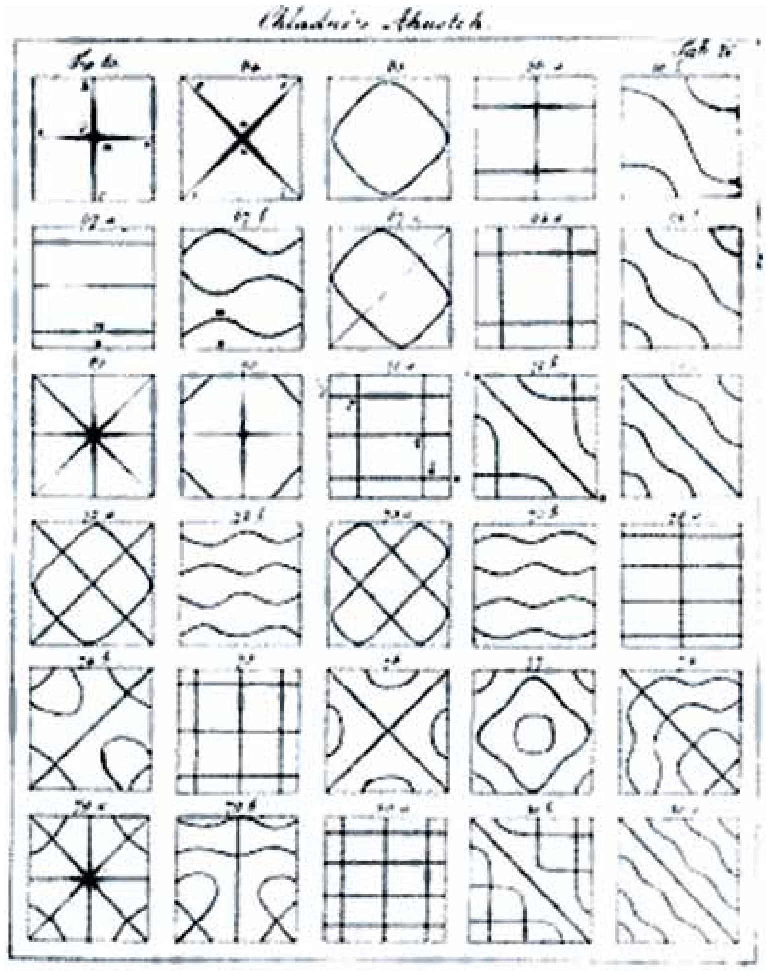
—*David Shavin*

1. This is as far as this brief summary will go. For a much fuller development, see Peter Martinson's 2008 "Quadratic Reciprocity." Otherwise, a useful chart (the one in color) may be found at: https://en.wikipedia.org/wiki/Quadratic_reciprocity.

2. Lyndon LaRouche, "The State of our Union: The End of our Delusion!" *EIR,* Aug. 31, 2007. Page 81. Available at Amazon: https://www.amazon.com/State-Our-Union-End-Delusion-ebook/dp/B01N2ZRDVL/ref=sr_1_1?s=digital-text&ie=UTF8&qid=1518549427&sr=1-1&keywords=The+State+of+Our+Union.+The+End+of+Our+Delusion.



Vibrations created on a Chladni plate with a violin bow produce nodal lines in sand.



From an 1802 edition of Chladni's Akustik.

musical invention, the glass armonica.[9] Hence, one can view Chladni as an intellectual grandson of Franklin both scientifically, through Lichtenberg, and musically, through Forkel.[10]

Chladni drew the sand patterns that arose from drawing a bow across the edge of the plate.

Chladni published his *Discoveries in the Theory of Sound* in 1787, and displayed his experiments in various cities in Europe.[11] In 1793, he spent a couple of months with Lichtenberg discussing electricity and acoustics. Further, his visit resulted in opening up Paris to Chladni. Fortunately, there had been a recent fireball in the sky over Göttingen in 1791, but the received view of the event—and, in general at that time, of meteors—was that they had to be the effluvia from volcanos. It seemed outlandish that rocks would be flying down from outside the Earth. Lichtenberg had Chladni spend time in the Göttingen library analyzing reports of various sightings, and computing trajectories, so as to prove the unearthly origin of meteors. Chladni's results, in his 1794 "Eisenmessen" report ("On the origin of the Mass of Iron… and other Ironmasses"), was not immediately accepted. But, in 1803, when the French Minister of the Interior commissioned the physicist, astronomer, and mathematician Jean-Baptiste Biot to investigate the recent meteor shower over L'Aigle, Biot confirmed Chladni's analysis—opening up an audience for Chladni in France.

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9. Chladni created his instrument, the "euphony," by replacing Franklin's rotating hollow glasses with tuned, cylindrical glass rods tuned at different pitches. Instead of rubbing the glasses with one's fingers, the rods could be initiated by pressing keys and, also, were more reliable in the resultant tone.

10. Of note, Chladni's work was most intensively studied and developed by the Weber brothers, Wilhelm and Ernst. Riemann, who was fascinated with his time spent in Wilhelm Weber's laboratory, furthered both Chladni's and Weber's work in acoustics with his famous "Shock Wave" paper of 1859 ("On the Propagation of Plane Air Waves of Finite Amplitude"). Riemann's student, Eugenio Beltrami, continued this tradition with his work on laminar flow.

11. When in Weimar, Chladni impressed Goethe, who proceeded to study his *Die Akustik*. Goethe reported to Schiller: "Doctor Chladni has arrived and brought his complete *Acoustics* in a quarto volume. I have already read half of it and shall give you a somewhat agreeable oral report on its content, substance, method, and form."

IV. Chladni's Harmonics: Excites Germain, Depresses LaGrange

In 1808, Gaspard Monge introduced Chladni's research to the Institute of France, where Chladni performed his experiment for its Class of Physical Sciences and Mathematics. All factions were provoked by what they saw, and Pierre-Simon Laplace arranged for Chladni, in February 1809, to repeat the presentation for Napoleon. Among those present were Biot, Felix Savart and Alexander von Humboldt. Chladni wrote that there was an awareness, including by Napoleon, that "one is not yet able to apply a calculation to areas curved in more than one direction"—or what can be described as the "arched-violin" problem—and Napoleon called for the matter to be made the subject of a prize contest.

Ernst Chladni

The reigning French authority in mathematics, Joseph-Louis Lagrange, declared that the known mathematics was not capable of accounting for the harmonic patterns displayed by Chladni. Sophie Germain reported that, prior to Chladni's visit, she had studied Chladni's works, but had been discouraged by Lagrange: "As soon as I learned about M. Chladni's first experiments, it seemed to me that analysis could determine the laws that govern. But I chanced to learn from a great geometer [Lagrange] whose first works had been devoted to the theory of sound, that this problem contained difficulties that I had not even suspected. I stopped thinking about it. Seeing M. Chladni's experiments during his stay in Paris excited my interest anew."

V. Harmony—The Attractive and Repulsive Actions of Molecules?

The Laplace/Lagrange faction formulated the prize contest called for by Napoleon according to their own ideological constrictions: Mathematical equations were to be developed that would account for the harmonic patterns of Chladni, but the equations should stem from the linear foundation established by Leonhard Euler's investigation of a vibrating bar. That is, entrants to the contest were to master Euler's treatment of a one-dimensional vibration, and then build upon that to account for the two-dimensional plate. They simply ignored the fact that Chladni had already shown, experimentally, that Euler's formula for the vibration rate, even for the one-dimensional case of a rod vibrating back and forth, was incorrect. (Chladni also showed that Giordano Riccati's correction of Euler was correct.[12]) Laplace's faction thought they had a champion for their cause, who could cook the numbers in their favor—Laplace's protégé, Siméon Denis Poisson.

In 1807, two years prior, Laplace had assigned Poisson the job of providing a mathematical cover for Laplace's defense of Isaac Newton. Empirical measurements of the speed of sound waves through air had refused to obey Newton's theory. Laplace, as recounted by Biot, had manufactured an "ingenious explanation of this difficulty by attributing the acceleration of sound to changes in temperature experienced by the particles of air as they condense and dilate." Poisson was supposed to buttress Laplace's defense of Newton by working out the mathematics of this model. The effort failed, but the intent of this faction was clear. The 1809 contest on Chladni's harmonics was to pose yet another opportunity to defend the Newtonian program of building up from fundamental particles and imputed forces, and then cooking the numbers to justify the defense.

However, none of their faction could actually generate a mathematical accounting that cohered with the harmonic patterns. Fortunately, Germain was never trained deeply enough in the technical manipulations of their faction, and it worked to her benefit. Though her entry was hampered by having to couch matters in the "Euler"-ian terms of the contest, she had the charming

12. Riccati's correction of Euler is found in his *Treatise on Elastic Fibres*. Of some interest, Riccati had also composed an "Essay on the Counterpoint Laws," and his musical collaborator, Andrea Luchesi, was the Kapellmeister in Bonn during Beethoven's youth.

insight to construct her analysis around the interaction of the two principal (maximum and minimum) curvatures of the Chladni plates in action.

Laplace, in a note to his 1809 "Memoire" on the subject, articulated the Newtonian ideology standing in the way, in his dismissal of an approach based upon the physical curvature: "In order to determine the equilibrium and movement of an elastic, naturally straight lamina that is bent into an arbitrary curve, it has been assumed that at each point its stiffness is inversely proportional to its radius of curvature. But this law is only secondary and derives from the attractive and repulsive actions of molecules, which are a function of distance."[13] For their faction, everything stemmed from fundamentally unknown hard balls interacting by means of a fundamentally unknown attraction or repulsion.

A letter drafted by Germain records her thoughts at the time: "But by far the greatest obstacle to the progress of science and to the undertaking of new tasks and provinces therein is found in this: that men despair and think things impossible... [I do not see] any strong objections to my theory other than the improbability of having it meet with justice. I fear, however, the influence of opinion that M. Lagrange expressed. Without doubt, the problem has been abandoned only because this grand geometer judged it difficult. Possibly this same prejudgment will mean a condemnation of my work without a reflective examination...."

The lawful result was that Poisson, and anyone else following the lead of Laplace, could not even formulate a presentable entry. By the 1811 deadline, Sophie Germain was the only entrant. She was denied the prize, as she had an insufficient grasp of the differential calculus. The contest was renewed, and two years later, with still no other entrants, she was given an honorable mention. Then, finally, she was awarded the "prix extraordinaire" for the 1815 contest, for her "Memoir on the Vibrations of Elastic Plates." Even then, the committee grudgingly admitted that her general equation had accounted for the Chladni harmonic patterns rather well, but added the disclaimer that they could not endorse her analytical method. They stated: "The differential equation given by the author is correct [in predicting the harmonics] although it has not resulted from the experimental demonstration." That is, she had not "built up" the mathematics from the hard facts, but rather had worked out her general equation from her physical hypothesis. Germain understood what was going on and posed the pointed followup question to the committee: Since her general equation came from her hypothesis regarding the principal curvatures, was that also incorrect? Her motto she had chosen to head her 1815 submission was from Virgil: "Fortunate is one who is able to know the causes of things."

The contest regarding Chladni's strange and beautiful harmonic patterns was now put aside. The Academy did not publish her prize-winning paper. The approach Germain had taken in her paper was largely ignored; and for Germain's last sixteen years, she was more tolerated than taken seriously. This year, 1815, was the beginning of the Restoration in France, when the Bourbon dynasty was re-established. The Ecole Polytechnique lost the leadership of Gaspard Monge and Lazare Carnot, and the reign of Augustin-Louis Cauchy over French science began. Over the next fifteen years, that reign would keep Germain too isolated, and would actively suppress the work of the other "poet-mathematician" students of Gauss's *Disquisitiones Arithmeticae*, the young geniuses, Niels Abel and Evariste Galois.

VI. Gaussian Curvature and Industrial Banking

We shall cite one provocative case as to what might have been, had Germain been able to benefit from any normal scientific exchanges. Just as Germain, in 1815, submitted her prize-winning paper on Chladni, a student of Monge at the Ecole, one Benjamin Olinde Rodrigues, attained his doctorate. He had developed tools in the rigorous treatment of intrinsic curvature that would have greatly benefitted Germain. His intrinsic curvature and his "total curvature" were more famously and thoroughly developed a decade later by Gauss (in his 1827 "Theorema Egregium" and his "Disquisitiones generales circa superficies curvas"), and is now referred to as "Gaussian curvature." Even allowing for her use of the prevailing extrinsic measurements of curvature, Germain's weakness in her treatment of the unified measurement of the maximum and minimum curvatures of the doubly-curved surface was her adoption of the arithmetic mean (and not the product) of the two curvatures. However, Germain would not hear of Gauss's developments on curvature until 1829; and in the reaction of 1815, Rodrigues, a Jew from Bordeaux, had no career in

13. Of note, in 1814, Lazare Carnot weighed in, promoting a memoir by one Paul Réné Binet, who had cited Lagrange's rectilinear formulation as failing to take account, even in the case of the one-dimensional vibrating bar of Euler, of the more complex torque component.

mathematics open to him—and it appears that Germain was not able to benefit from his work either.

Even so, it were still possible that Germain and Rodrigues could have collaborated outside of the Ecole Polytechnique or the Academy of Sciences. Blocked from a teaching position, Rodrigues and his brother set up a "national banking" salon that might have overlapped with Germain's father, himself a former directeur of the Banque de France. The salon included: Jacques Laffitte, also a former directeur of the Banque de France, and a proponent of industry and railroads; Vital Roux, a regent of the Banque de France, whose pamphlets agitated for directing credit toward industry; and Emile and Isaac Pereire, cousins of Rodrigues. (The technical consultant at the Pereires' industrial concern, Michel Chevalier, was Galois' friend. His brother, Auguste, was the one who saved Galois' works.) Rodrigues himself would be key in the development of France's first operating railroad, similar to the role in Prussia of August Crelle, the sponsor of Niels Abel. It were quite possible that Germain's father socialized with these industrial- and science-connected bankers, however no link with Rodrigues' national-banking salon has yet been established—and no evidence of Rodrigues' introduction of the intrinsic measurement of curvature detected in Germain's work.

VII. Germain and the New, Young Poet-Mathematicians

With the exception of Bernhard Riemann, Germain had more direct interchange with Gauss than the other three leading students of Gauss's *DA*. And she was more centrally placed than any of the other four, to potentially serve as a focal point for their work. Dirichlet, Abel and Galois came to, or were in, Paris between 1822 and 1832, where Germain was the leading student of Gauss over the last two decades. However, as a woman, she did not have proper standing amongst the scientific community to make such a collaboration naturally develop. Dirichlet, Abel and Galois would find Paris a forbidding and hostile environment. Only Dirichlet survived the experience.

Dirichlet was in Paris from 1822 to 1827, and both he and Germain used Gauss's *DA* to work on Fermat's Last Theorem, and on the problem of the quintic—that is, why algorithms, or generalized mechanical solutions, of algebraic equations broke down at the fifth power, as if running up against an unseen barrier. Beginning in May 1823, Germain was finally allowed to attend and listen to presentations of the Academy, being provided with tickets by the Academy Secretary, Jean-Baptiste Fourier. She probably attended Dirichlet's presentation on Fermat, made to the Academy in July 1825. She herself had prepared a twenty-page memoir on Fermat's Last Theorem several years earlier, telling Gauss in 1819 that his *DA* was the basis of her strategy. And two months after Dirichlet's 1825 presentation, Germain's correspondent, Legendre, presented his followup to Dirichlet to the Academy, one that included a mention of Germain's work. It seems probable that Germain and Dirichlet would have met and discussed their work, but no record of such is known.

What is known is that in May 1825, Germain finally meets, in person, with Guglielmo Libri, an Italian student of the *DA* who had been in touch with Germain since 1819, but now had travelled to Paris. Libri presents his work before the Academy on June 13, 1825, several weeks prior to Dirichlet. Libri would be Germain's closest collaborator between 1825 and 1831. He was to become both the author of her biography and the preserver of her work. Libri had a very colorful life, one that is beyond the scope of this series.

Germain ended up, in the 1820s, having to publish on her own her works on the Chladni plates. Her papers submitted to the Academy were not published, nor even provided a courtesy review.[14] It wasn't until 1828 that an article of hers was finally published by a scientific journal—*Annales de chimie et de physique*, edited by François Arago and Joseph Louis Gay-Lussac, had the honor.[15] There, on the subject of the dynamics of elasticity (of laminar surfaces, such as the Chladni plates), Germain reminded her readers of her overlooked approach, and explained why Siméon Poisson's much-promoted approach was inadequate, and referred the readers to her self-published 1826 report on elastic surfaces, one that Cauchy had suppressed at the Academy.

The infamy of Cauchy in actively working to crush Niels Abel and Evariste Galois, the youthful geniuses of Gauss's *Disquisitiones Arithmeticae*, was covered earlier in this series.[16] However, it was in the weeks immediately prior to Cauchy's burial of Abel's work, that

14. Germain's 1824 "*Effets dus a l'epaisseur plus ou moins grande des plaques elastiques*" was assigned for review, but was buried. Her 1825 paper assimilating recent developments in acoustics (including those of Charles Wheatstone) was simply ignored.

15. See Germain's 1828 article at https://babel.hathitrust.org/cgi/pt?id=hvd.hx3d vx;view=1up;seq=131

16. See footnote 1.

Cauchy was already at work on Germain's case. On July 7, 1826, Germain sent a long letter to Cauchy, reviewing how developments since 1815 only strengthen her physical-hypothesis approach, and undercut the Newtonian approach. There, she refuted Poisson's molecular explanation, explaining further that, regarding elastic bodies, imbedded assumptions about the molecules are useless and even harmful.

Fiinally, she took apart Felix Savart's experiments, even though Cauchy would still use them as support for his own work. Cauchy relied upon Savart, who had won his entry into the Academy circles a few years earlier with his bizarre, flat violin, one designed in the shape of a trapezoid! Here, a picture is worth a thousand words—the trapezoidal violin was emblematic of the problems with this faction. Savart's obsession for maximizing the vibrations of small parts ended up in a big failure.[17] Germain wrote of the celebrated Savart: "This Monsieur Savart would have been able to help me a lot if he wanted to use the kind of sagacity with which he is endowed with good experiments on curved surfaces."[18] Cauchy did acknowledge receipt of both Germain's letter and the memoir that she had submitted to the Academy, but no more. Fourier, the Academy's Secretary, told Germain that Cauchy was assigned to report on her memoir, but Cauchy never gave that report. Within weeks, with the submission to Cauchy of Abel's magnum opus, Cauchy would graduate to become the infamous serial abuser of Gauss's students.

By eliminating the upper and lower chambers of the violin for a simpler trapezoidal box, Savart aimed to maximize the amount of vibrations at the surface. He only had to sacrifice the beauty of a lased, bel canto *sound.*

VIII. Germain's Swan Song—Her Last Two 'Gauss' Papers

Niels Abel died in 1829 at the age of twenty-six. In 1830, his mentor, colleague and publisher, August Crelle, visited Germain. Crelle came from Berlin, on a mission in collaboration with the Humboldt brothers, to study what methods the Ecole Polytechnique had used to build up a national science program. Crelle was impressed with Germain and agreed to publish her works in Berlin. Sophie raced against time, and debilitating pain, as she was dying from a cancer detected the previous year. The two works that she chose to leave the world were both based upon Gauss: a memoir on the curvature of surfaces and a summary of the original material that she had sent Gauss in 1804.[19]

The previous year, in early 1829, Gauss had instructed his student Bader to deliver a copy of his *Theoria residuorum biquadraticorum* [Theory of Quadratic Residues] to Germain. She reported back to Gauss, March 28, 1829: "I have read, with great pleasure, your memoir on biquadratic residues, which this young scientist has given me on your behalf." Germain then briefed Bader on her own work, which prompted a discussion of Gauss's latest work on curvature. Bader brought out "the learned memoir in which you compare the curvature of surfaces to that of the sphere (Gauss's 1827 "Disquisitiones generales circa superficies curvas")... [General Investigations of Curved Surfaces.] I cannot tell you, Monsieur, how astonished, and at the same time, how satisfied I was in learning that a renowned mathematician, almost simultaneously, had

17. A real violin is designed with an upper and lower chamber, on the model of the head and chest cavities used in "bel canto" singing. The dynamics involved in the coupling of the resonances of the two chambers is not built up from percussive interactions of hard bodies. See the author's unpublished 2010 report, "Leibniz's Dynamics & Stradivari's 'Bel Canto' Violin Breakthrough."

18. For her own experiments, Germain had employed a skilled mechanic named Mons Moulfarine, to make thin glass plates of varying curvatures and thicknesses.

19. In her work on curvature, she had employed a formula for the radius of curvature of an oblique surface that had been developed by the student of Monge, Charles Dupin—covered earlier in this series as a model for Edgar Alan Poe in his treatment of Galois. (It is unclear what Germain knew of Dupin, as she attributed his formula, mistakenly, to Jean Baptiste Meusnier.)

the idea of an analogy that seems to me so rational that I neither understood how no one had thought of it sooner, nor how no one has wished to give any attention to date to what I have already published in this regard."

But Bader and Germain don't have enough time to get everything resolved. Bader does not have a duplicate copy of the curvature paper to leave with Germain, so she has not been able to fully digest Gauss's work. (Apparently, in determining the radius for her referent sphere, she was still employing the arithmetic mean of the minimal and maximal curvatures.) She tells Gauss of Poisson's objections to her approach: He was "relying on Euler's discussion of the infinite number of different curves obtained from the intersections of different planes passing through a given point of the surface" and he "had thought that I had not sufficiently established the choice of principal curvatures.... I am in the process of proving, in a superior way relative to what I have published previously in this regard, that whatever be the shape of the element of the surface, that is to say, whatever be the manner in which the curvature of the element is distributed about the point of tangency, the force that would be employed to destroy the curvature of this element remains constant.... I regret ... not being able to submit to your judgment a multitude of ideas that I have not published and that would take too long to write out."

The French government highlights the life and work of Sophie Germain in this 2016 postage stamp.

In her last few months, Germain did have the satisfaction of finally seeing her two works published in a major scientific journal. Her work on curvature was composed in 1830, ironically, during the few days of the turmoil of the July Revolution. Perhaps a coincidence, but once again—as when during the violence of July 1789, the thirteen-year-old girl found sanctuary in her father's library and found the genius of the endangered Archimedes—Sophie accessed her inner voice.[20]

IX. The Science of 'Different Modalities'—Transcendental Music, Bach and Gauss

Sophie Germain died on Monday, June 27, 1831.[21] In her last letter to Libri, a month before her death, she expressed her conviction of the unity of art and science: "Ah! No doubt, the sciences, literature and fine arts were born of one and the same sentiment. They reproduced, according to the means that are the essence of each of them, copies of their constantly renewed innate style, a universal type of truth, that is so strongly imprinted in superior minds." What informed, drove and sustained Germain, during her long battle for the beauties of Chladni's harmonic pictures, was her music—and her unwillingness to betray the beauty of the inner soul.

The role of music in Germain's life is simply not mentioned in the accounts of her approach to the Chladni plates, though it is painfully obvious, during the long battle for her approach, that she would not allow the harmonic patterns to be reduced to things that go bump in the night. In 1833, Germain's nephew, De Lachevardière, published her thoughts on these matters in a work entitled *Considerations generales sur l'état des sciences et des lettres aux differentes époques de leur culture*. Libri explains about this posthumous work, that "...among her papers have been found some very subtle philosophical reflections, for she was actively occupied with metaphysics, which she claimed was the source of the true philosophical spirit. She thought very little of diverse philosophical systems.... [She had] an ability

20. Her biographer Libri puts it: "When the revolution of July broke out, she took refuge in her study as she had during that of '89; it was during the week of fighting that, taking up and developing further some old ideas, she wrote her '*Memoire sur la Courbure des Surfaces*,' which appeared in the *Annales* of M. Crelle of Berlin."

21. In Paris, seventeen days later, Galois was arrested and jailed, leading to his death at the age of twenty. Though barely fifty-five, Germain actually lived longer than her four, compatriot, Gaussian "poet-mathematicians." In her last two years, Sophie lamented the death of Abel and the unappreciated genius of Galois.

… to reconcile similarities between the physical order and the moral order, which she regarded as subject to the same laws."

Amongst her extended reflections on the gap between the scientific pursuit of truth and the emotional level of her culture, Germain argues for an underlying unity of beauty and truth. In her words:

"The oracles of taste and the dictates of reason are similar; order, proportion, and simplicity never cease to be intellectual necessities. Their subjects are different, but the judgment is constantly based on the same universal type, which belongs equally to the beautiful and to the truth."[22]

"A trait of genius … in the sciences, in the fine arts, or in literature, all have the same effect of making us happy for the same reason: they reveal to us all sorts of relationships that have escaped us. We are suddenly transported into a high region where we discover a new ordering of ideas and of emotions."

In the work, Germain expresses a deep-seated fear of the outbreak of violent emotions as displayed in the 1789-94 turmoil (and, possibly, also from the events of July 1830). She argues that leaders have not been sufficiently developed to deal with revolutions. Decent leaders in normal times are clever enough. "In times of crisis, however, it's something else. Circumstances become pressing; we must know how to make prompt decisions; we also often need courage, and courage is not necessarily a quality of the clever man. Society runs a thousand dangers which are as difficult to avoid as they are to predict." How might we unite genius and courage? It turns out that it is a too rarely-exercised transcendental power that is required, one that Germain knows well from the mastery of modalities displayed by Gauss's *DA*.

Then, somewhat surprisingly, Sophie singles out the missing ingredient in France and in Europe—the musical equivalent of what she has heard in Gauss's "poetry." She argues that, while music has the universal power of being able to "strike at the truth for the least educated man," it also takes work to master the harmonic whole. This necessary work has been avoided "because of the prejudice that separates music from the field of intelligence." The educated might achieve a certain level of literacy, but it lacks rigor. But "with respect to music, things are quite different." While some of the educated may even come to appreciate many effects in music, this is still below the mastering, e.g., the overlooked genius of Bach's well-tempered system. "Today, we no longer understand what history has given us through the teaching of the different modalities. Therefore, how could we ever be conscious of this when music is only considered as the art of caressing the ear? How could music be the object of serious attention when it is reduced to such an exclusive use? … Music is completely metaphysical."

X. In Conclusion: Genius and Happiness—Or, Playing Ping-Pong with the Stars

Sophie Germain knew quite well what it was like to have her scientific work treated as the curiosities of a woman caressing the ear of the scientific establishment. That did not deter her from her mission.

She heard in Gauss's "poetry" a revival of Bach's unified development of the different modalities, and thought her culture was suffering from the retreat from Bach's level of science. Gauss's treatment of the hidden truths uniting the modalities was a pathway for civilization to train its leaders to deal with revolutionary stresses and revolutionary solutions.

Gauss's uniquely rigorous examination of what seem to be the completely familiar 1, 2, and 3's of arithmetic, uncovers profound insights as to how the human mind works when it ventures to order the world. Sophie Germain thinks this is key to the pursuit of happiness: "A trait of genius … in the sciences, in the fine arts, or in literature, all have the same effect of making us happy for the same reason: they reveal to us all sorts of relationships that have escaped us. We are suddenly transported into a high region where we discover a new ordering of ideas and of emotions."

Lyndon and Helga LaRouche think that Americans can still engage in this pursuit of happiness, should they forgive themselves for a few decades of becoming small and petty, and allow themselves to seize the historic opportunity of the great infrastructure projects of the Belt and Road. We would discover "all sorts of relationships that have escaped us" and find ourselves "suddenly transported into a high region where we discover a new ordering of ideas and of emotions."

22. Pierre Beaudry has kindly provided the translations of key portions of Germain's *Considerations generales*. … I told him of my suspicions that evidence of Germain's reliance upon beauty and music for her scientific work might lie within the work, and he immediately tracked down the relevant content.