
III. Lyndon LaRouche's Original Work

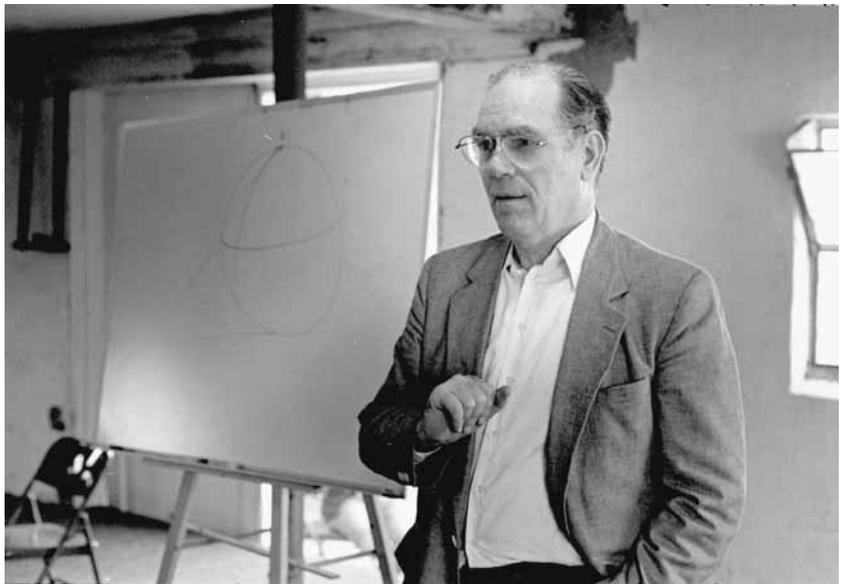
November 2, 1988

Beethoven as a Physical Scientist

by Lyndon H. LaRouche, Jr.

Preface to the first publication, in the EIR of May 26, 1989

Executive Intelligence Review is pleased to publish in a preliminary version the following paper, originally written by Lyndon LaRouche on Nov. 2, 1988. We believe that the ideas here are so crucial to the survival of our civilization, that there should be no further delay in making them widely available. Not all of the illustrations originally called for by Mr. LaRouche have been sufficiently researched to be publishable at this time, and this is partly due to the fact of the author's trial and unjust imprisonment as of Jan. 27, 1989, which has made it difficult for him to personally guide the scientific work at the necessary pace.—The Editors.



EIRNS/Stuart Lewis

The author gives a class in geometry and physical economy in 1985, at the barn of Ibykus Farm in Loudoun County, Virginia.

Like a skilled cabaret mimic, purporting to mimic well-known public figures of politics and entertainment, significant numbers of professional musicians have produced short improvisations which audiences might recognize as parodies of a Mozart, Beethoven, Schubert, and so forth. Yet none of them could have produced a composition which might be confused with an actual work of those or other classical composers.

Thus, the ability to compose artistic statements in the literate language of music common to all classical composers, is virtually as lost today as the ancient Egyptian was lost until the discovery of the Rosetta Stone. Fortunately, the almost language of classical composition can be reconstructed, and the result proven

conclusively to be an accurate one. This accomplishment is one of the leading projects which this aging writer is determined to see completed before he “shuffles off this mortal coil.”

It is the solution to that problem, which is identified here.

The timing of the appearance of this report reflects the recent progress of work on the subject of Eugenio Beltrami's crucial discoveries respecting the negative curvature of physical space-time. Crucial proof of Beltrami's corrective supplement to Riemann curvature renders intelligible to a much deeper degree, the otherwise empirically demonstrable principles of composition of classical music. It is therefore the appropriate

time to publish a summary account of key points of progress toward the goal of defining rigorously the principles employed by such composers as Bach, Mozart, and Beethoven.

Excepting the indispensable function of advanced geometries, for rendering musical principles truly intelligible, most of the pieces of evidence to be brought together are already well-known among relevant classes of scholars and fine-arts professionals.

The principle of natural beauty, for example, has remained constant in classical fine arts, including music, since before the time of Plato in ancient Greece. It is also more or less well-known among relevant scholars and professionals, that the mere imitation of natural beauty, as we find it common to the morphology of growth and function of living processes, is not sufficient to class a painting or song as a work of classical fine art. While remaining ever-faithful to demonstrable harmonic principles of natural beauty, the work of art must incorporate and radiate that special quality of mental life which sets mankind apart from, and above the beasts: the unique potential of persons, the development of the creative powers of the human mind.

Respecting classical musical composition as such, most of what is verified as knowledge of the performing techniques of the eighteenth and early nineteenth centuries, is either valid beyond doubt, or, at worst, not untruthful as far as it goes. The breakdown in knowledge occurs wherever modern professional musicology, as taught in the relevant classrooms of universities and music conservatories, is confronted with a topic which bears upon presenting the method of composition commonly employed by classical composers in the form of an intelligible, and verifiable principle.

The most immediate, practical objective of this undertaking, is both to enhance the powers of the best classical performers, and to enable amateurs among their audiences to read a classical score as the composer intended it should be read. More broadly, the purpose is to develop means by aid of which the beauties of the greatest classical fine arts might more easily beautify and otherwise enrich spiritually the lives of the vast numbers of persons suffering the frightening pangs of spiritual want, experienced by all who lack enrichment of this aspect of their living.

Had a student of music been reared in the relevant places in western Europe, during the period from

Brunelleschi's Florence, through as late as 1849, he or she would have learned to speak the language of classical musical composition in a literate degree, and with comprehension of principles involved as intelligible ones. Since the language of classical poetry and music has been lost, in that sense, over a period of more than a hundred years, to adduce the same principles of literacy today is far more difficult than would have been the case in those earlier times.

For reasons to be indicated here, the required reconstruction can not be accomplished without reference to principles of physics associated with the work of such as Gauss, Riemann, Beltrami, and Georg Cantor, during the mid-portion of the nineteenth century. It is certain that Ludwig van Beethoven did not know his principles in that form of representation; yet, in a relevant sense, he mastered those principles very well, as we, today, can show beyond all reasonable doubt.

Hence, it is not only permissible to refer to Beethoven as a physical scientist; under today's circumstances, it is more or less mandatory that we do so.

On this account, the author has adopted the view, that no statement of principles of composition is truly intelligible and also verifiable musically, unless it enables professional musicians to reach a point of breakthrough to deeper understanding of the most astonishing episode in all classical composition, the exemplary compositions of Beethoven's last period of work. The last quartets, including the "Great Fugue," beginning with Opus 127, should be singled out as the most concentrated expression of this most challenging episode in the entire history of music to date.

The proof that any progress in unravelling this singular episode is valid, depends upon showing that Beethoven's principles during that last period of his work are coherent with what we may adduce from study of the most relevant work of J. S. Bach. For musical-historical reasons, the simplest way to establish that connection is the influence of Bach's "Musical Offering" on the leading work of Mozart, Beethoven, Schubert, and Chopin, for example, later. (**Figure 1.**)

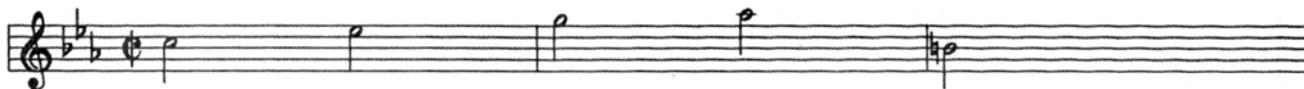
A re-examination of the work called "Bach's Art of the Fugue," must be considered from the standpoint of what the later work of Beethoven shows to be the deeper principles embedded implicitly in the Bach canons of polyphony, as later composer's treatment of the "Musical Offering" enables us to trace the connections most explicitly.

FIGURE 1

Ascending and Descending Sequences in Nine Related Musical Compositions

ASCENDING

J.S. Bach, *Musical Offering*, Ricercar à 3 voci



W.A. Mozart, Sonata for Piano in C minor, K. 457



W. A. Mozart, Quartet for 2 Violins, Viola, and Violoncello, K. 465 ("Dissonant")



The Singing of Classical Poetry

In physical science, except as we demonstrate the power to create physical states which have not existed in the known universe before man's artificing such states, every valid fundamental discovery in physics is premised upon what are termed "crucial experiments" showing a fallacy in reading of what nature has already accomplished. Such is the history of music.

With aid of development of what is known as the study of non-linear spectroscopy of optical biophysical processes, we are enabled to understand, as a matter of biological principle, whence certain characteristic and crucial features of the singing and hearing of a sung bel-canto scale pivotted on either Middle C at 256 cycles, or approximately midway between 256 and 257, a value almost precisely 42 octaves below the characteristic frequency of living DNA. (Table 1.)

However, earlier, excepting the influence of the physics work summed up by Johannes Kepler, the development of classical principles of singing and musical polyphony generally was premised upon the empirics of what we reference today as the "bel canto" trained singing voice.

The classical, man-made musical instruments were designed on the basis of physical principles adduced

from empirics of the "bel canto" form of singing voice, and were constructed in conformity with the assumption of the value of Middle C as exactly, or nearly C=256. The different species of stringed instruments and wind instruments were treated as an extension of the polyphonic choir composed of various species of singing voices, and the principles of vocal polyphony were imposed upon the instrumental voices, to the effect that the very notion of an "instrumental interpretation," differing from a vocal one, is a bestializing absurdity contrary to the most elementary principle of classical composition.

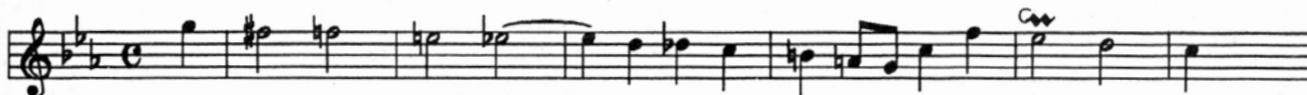
The root of music is the singing of classical poetry. The polyphonic singing of classical poetry is uniquely the origin of the tonal construction of the well-tempered musical scale, and of the metrical structure of musical composition. (The attribution of the metrical structure of music to an origin in the dance, as that hoaxster Richard Wagner insisted, is based upon Nietzschean aesthetics, as the Romantic and Modernist dogmas trace the origins of music to the eroticism of the phrygian cult of Nietzsche's Satan-Dionysos.)

As early as Vedic hymns located by included solar-astronomical evidence to no later than 4,000 B.C., any classical poem is also a musical score. The physical

FIGURE 1 (continued)

DESCENDING

J.S. Bach, *Musical Offering*, Ricercar à 3 voci



W. A. Mozart, Sonata for Piano in C minor, K. 457



W. A. Mozart, Quartet for 2 Violins, Viola, and Violoncello, K. 465 ("Dissonant")



basis for this well-known and practiced fact, in the singing of classical poetry, was first explored rigorously by Leonardo da Vinci, who was first to document the changes in pitch inherent in shifting enunciation from one vowel, or consonant-inflected vowel, to another. [Table 2 illustrates this point for the vowels alone, without consonant inflection, in a first approximation. **Figure 2** is a table of vowels in combination with various consonants prepared by Leonardo da Vinci.]

For example, during the course of the nineteenth century, in Germany, these studies by Leonardo were combined with bel-canto principles of voice production, to provide the program for training of both singers and classical dramatic actors.

The differences in intonation of the vowels, as each literate form of language defines the intonation somewhat differently, does not define an absolute, fixed tone, for that vowel, or consonant-inflected vowel. What is fixed is the relative value of the intervals separating these intonations from one another.

In both classical poetry, and classical song based upon classical poetry, the absolute value of the intonation of a vowel or consonant-inflected vowel shifts as the reference tone for C set to, for example, Guido's

"ut" [see Table 2, Guidonian singing scale] changes. This is easily demonstrated by reference to the practice of placing a sequence of tones in the bass part, as pedal-point, and using those tones as the reference tones for the choices of absolute tone-values corresponding to the selected intervals among the indication vowels and consonant-inflected vowels.

Thus, the prosodic determination of the intervals now interacts, in both classical reciting of poetry, and classical song, with an harmonically ordered sequence of tones in the actual or implicit pedal-point. These pedal-point sequences are either directly the Kepler intervals, or consistent derivatives of those intervals, such as the well-tempered octave-scale itself: octave, fifth, fourth, major third, minor third. In constructive geometry, this interaction is represented as doubly-connected action.

However, actual musical composition must reference immediately two additional degrees of such interaction, confronting us with an elementary musical domain which is, respectively, triply-connected, and quadruply-connected.

From the human singing voice, we have the following consideration. Each species of singing voice is characterized, in each of all cases, by a set of unique

FIGURE 1 (continued)

Ascending and Descending Sequences in Nine Related Musical Compositions

ASCENDING

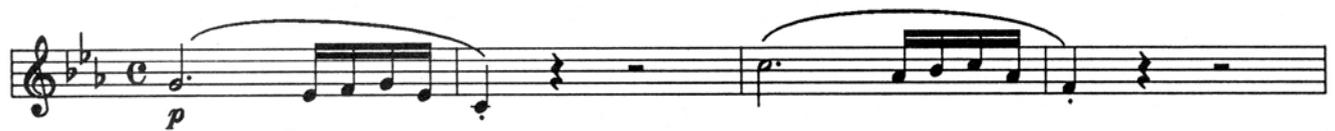
W. A. Mozart, Fantasy K. 475



L. v. Beethoven, Sonata for Piano in C minor, Op. 13 ("Pathétique")



L. v. Beethoven, Sonata in C minor for Violin and Piano, Op. 30, No. 2



L. v. Beethoven, Sonata for Piano in C minor, Op. 111



F. Chopin, Sonata for Piano in C minor, Op. 4



F. Schubert, Sonata for Piano in C minor, D. 958 (Posthumous)



FIGURE 1 (continued)

DESCENDING

W. A. Mozart, Fantasy, K. 475



L. v. Beethoven, Sonata for Piano in C minor, Op. 13 ("Pathétique")



L. v. Beethoven, Sonata for Piano and Violin in C minor, Op. 30, No. 2



L. v. Beethoven, Sonata for Piano in C minor, Op. 111



F. Chopin, Sonata for Piano in C minor, Op. 4



F. Schubert, Sonata for Piano in C minor, D. 958 (Posthumous)



intervals of the well-tempered scale, referenced to C=256, at which the quality of the singing voice must change (or incur damage to the singing voice). (See **Figure 3**).

So, when the classical composer sets a classical poem to song, the composer must choose, in the simplest case, a definite species of male or female singing voice. He must choose a major or minor key-signature, such

TABLE 1
The Musical Scale and the Biological Spectra

Mitogenic radiation	42 octaves + F-341 Hz (200 nanometers)
Pure protein alpha helix	42 octaves + E-326 Hz (208 nanometers)
DNA	42 octaves + C-256 Hz (265 nanometers)
Protein complex	42 octaves + B-243 Hz (280 nanometers)
Vision (lower bound)	42 octaves + G-188 Hz (360 nanometers)
Register shift between ultraviolet and visible F—F-sharp	
Chlorophyll-a	42 octaves + E-158 Hz (430 nanometers)
Carotene	42 octaves + D-141.5 Hz (481 nanometers)
Photosynthesis action spectra	42 octaves + C-128 Hz (536 nanometers)
Vision peak	42 octaves + B-122 Hz (560 nanometers)
Cytochrome	42 octaves + B-flat-114Hz (595 nanometers)
Chlorophyll-a	42 octaves + A-flat-102.25Hz (660 nanometers)
Bacteria photosynthesis center 1	42 octaves + G-94 Hz (720 nanometers)
Register shift between visible and infrared F—F-sharp	
Bacteria photosynthesis center 2	42 octaves + E-80 Hz (850 nanometers)
Bacteria photosynthesis center 3	42 octaves + E-flat 75.5 Hz (900 nanometers)
Biosphere maximum radiation	42 octaves + C-64 (1,072 nanometers)

The key moments of biological processes range 42 octaves up from the F above middle C to the C two octaves below middle C, which is itself 40 octaves above C =256. All values are precise musical tones in cycles per second (Hz) plus 42 octaves. The initial experimental values in wavelengths are given in parentheses.

Source: Warren J. Hamerman, "The Musicality of Living Processes," *21st Century Science & Technology*, March-April, 1 989, p. 34. Reprinted by permission.

as C-major or C-minor, which causes the natural voice-register shift of the bel-canto-trained singer to change quality at some desired place in the vocal line.

Thus, as the table shows, a song written for a species of female voice might be transposed for another female voice, but not for a species of male voice, without losing something important in the classical composer's original musical intent. The distribution of register-shift intervals in the male voices differ from the distribution in the female voices.

In the case of vocal polyphony, using perhaps two different species of female voices (e.g., soprano and mezzo-soprano, or tenor and bass or baritone), the classical composer is confronted with some interesting problems, on account of the principle just referenced. The simplest canon, based on such a polyphonic combination of species of voices, incurs extremely fascinating requirements.

The intent of the poem as a poem, as read by the composer, must be served. Hence, the natural voice-register shifts of all of the voices must be treated accordingly.

This fascinating problem in the most elementary feature of the composition of the classical principles of polyphony, even in the form of the simplest canonical exercises, defines the most rudimentary features of the principles of well-tempered counterpoint.

These voice-registration considerations must be superimposed upon the doubly-connected domain identified above. In the language of constructive geometry, the most rudimentary classical counterpoint is already triply-connected.

In the cases of the instruments, we must expand the notion of counterpoint, without changing any underlying principle. We must treat each instrument as a species of singing voice, and view its intrinsic registral characteristics accordingly. The case of the duet between a classical soprano oboe, constructed with reference to C=256, and a true soprano or mezzo-soprano, is among the most beautiful phenomena in the repertoire, and thus beautifully instructive of the point we have just made. The same relationships, with differences duly noted, are relevant to all cases.

Yet, except as such triple-connectedness is applied to the interpretation of classical poetry for composition of song, we have not exceeded the domain of natural beauty. In all cases of exercises in classical counterpoint according to the principles of triple-connectedness, we have not yet bridged the distinction between the imitation of natural beauty and fine art.

In true classical fine art, whether in painting, architecture, city-planning, drama, poetry, or music, we must never violate adducible lawful principles of natural beauty in the composition as a unit-conception; yet, unless this is accomplished by adding something crucial, it is mere describing of nature, and, however useful that description might be, it is not yet a creative work, and hence not a work of classical fine art.

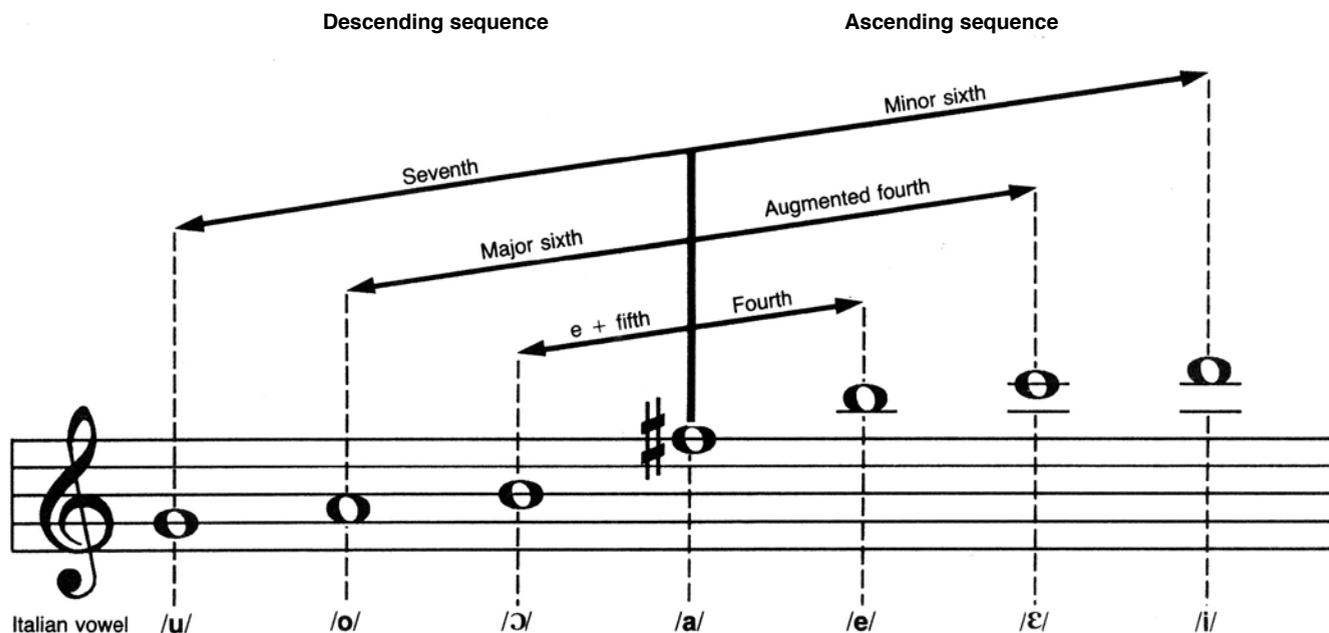
The essence of music, which carries it beyond a mere

description of natural beauty, is the use of the medium of natural beauty as the medium of expression of the creative powers of the individual human mind. These are the same powers represented by the generation of a valid fundamental discovery in physical science.

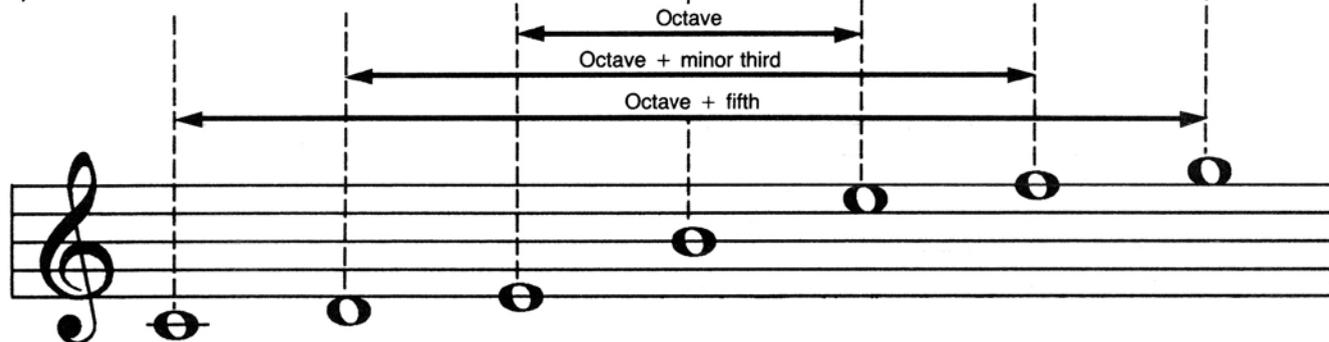
Most briefly, merely to identify the point: The characteristic of classical music, is the lawful generation of a combination of harmonic and metrical dissonances, typified in harmonics by the interval between C and F#.

TABLE 2
Vowel harmony: relative pitch of Italian vowels
 (based upon second formant, F²)

A) Transposed down one octave



B) Relative intervals with /u/ set at C=256 Hz



The musical quality of vowels in speech is lawfully determined. This chart shows the musical-interval relations among the seven Italian vowels based upon vowel quality. It is based upon recent studies of native Italian speakers and is derived from the second formant, or resonance peak, as measured for the different vowels in Hertz. The test data was based upon men's voices only. For clarity, the tones have been transposed down one octave in (A). In (B) the intervals have been transposed so that the /u/ vowel corresponds to the syllable "ut" (the first tone of the Guidonian singing scale) set at middle C of 256 Hertz.

FIGURE 2

Leonardo da Vinci's Chart of Vowels



Leonardo showed his interest in the apparatus of voice production in this sheet of studies which includes a detailed analysis of the tongue muscles, and a chart of vowels combined with various consonants (upper right corner).

F# is the tone at which the soprano voice must shift naturally to a higher register, and the tenor must pass from the second to third register, so. Thus, the soprano and tenor voices divide the octave referenced to C=256 into two tetrachords, as a study of the score of the relevant *urtext* of songs of Mozart, Beethoven, and Schubert illustrate the point.

The *relative* dissonance must never be reached arbitrarily, as irrationalist forms of composition, such as chromatic Romanticism, Modernism, and jazz do. It must never occur except as a necessary occurrence within the elaboration of polyphony according to lawful principles of classical counterpoint.

The essential feature of any classical composition as a work of art is the generation and resolution of such combined harmonic and metrical *relative* dissonances. Having posed the occurrence of such *relative* dissonances as a musical problem, the composer must resolve that problem. The composer must elaborate a process of contrapuntal development, as Beethoven does with his musical-compositional montages, which carries the process of composition as a whole to a resolution. At the end, the affirmation of the resolution imparts to the listener's mind the fact that a complete musical idea has been stated. At least, this is the case if the performance of the entire composition has been articulated to effect a kind of "long phrasing" which is

coextensive with the performance as an entirety.

This problem and its resolution, completed by the final sequence of tones of concluding affirmation, represents a musical idea. Nothing less does.

Among the best available, simpler illustrations of this process, is the surviving notes of Beethoven bearing upon the compositional pre-history of his Opus 106 "Hammerklavier" piano sonata. The fact on which we wish the reader to focus, is the enormous span of intellectual work which occupied Beethoven in successive revisions of the design of what became the opening thematic statement of that composition.

In effect, in this work, Beethoven was working backwards, from the definition of a musical idea, to shaping a germinal thematic statement, whose contrapuntal elaboration would lead to the generation of the musical idea through such a compositional process.

Later, in referencing the last stages of polishing of the completed composition, and in viewing the succeeding piano sonatas, Opus 109, 110, and 111, as a unit of compositional process output, we see Beethoven's repeated retrospective reference to the same musical idea which preoccupied Mozart before him, that of J. S. Bach's "Musical Offering."

By considering the Opus 106 in these enlarged terms of reference, we see more deeply. Note the prefixed amendment of the second movement of the 106, the *Adagio Sostenuto*. Note the last movement, its great double fugue. Compare the conclusion of these series of compositions, the Opus 111, with the first movement, and more, of Chopin's "funeral march" sonata.

It ought to be obvious that Beethoven's last quartets, beginning with the Opus 127, and including the Great Fugue Opus 133, must be treated as a unit-series of exposition of the same species of musical idea, in the same sense that the Opus 106, 109, 110, 111, must be viewed as a unit-idea series.

This sequence of unit-idea series, in Beethoven's last period of composition, beg comparison with a succession of stages of valid scientific revolutions. Each unit-series of compositions is much more than a specific musical composition; it is a musical scientific revolution, from which music must not turn backwards. Hence, the occurrence of these so emphatically in clusters of closely related compositions, even much more so than in Beethoven's earlier publishing practice.

In the simplest of all possible cases, an harmonic series corresponding to a creative developmental process may be referenced to no less than three notes, and

the same interval expressed by other sequences of notes. On this account, and expanded considerations, the minimal requirement of any classical musical composition is that it be defined as a quadruply-connected process of composition.

We mean that statement in the fullest sense of the topological implications of a such a statement. This brings our inquiry, implicitly, into the mathematical-physics domain of Gauss, Dirichlet, Weierstrass, Riemann, Cantor, and the great Beltrami.

Science and Music Are ‘Non-Euclidean’

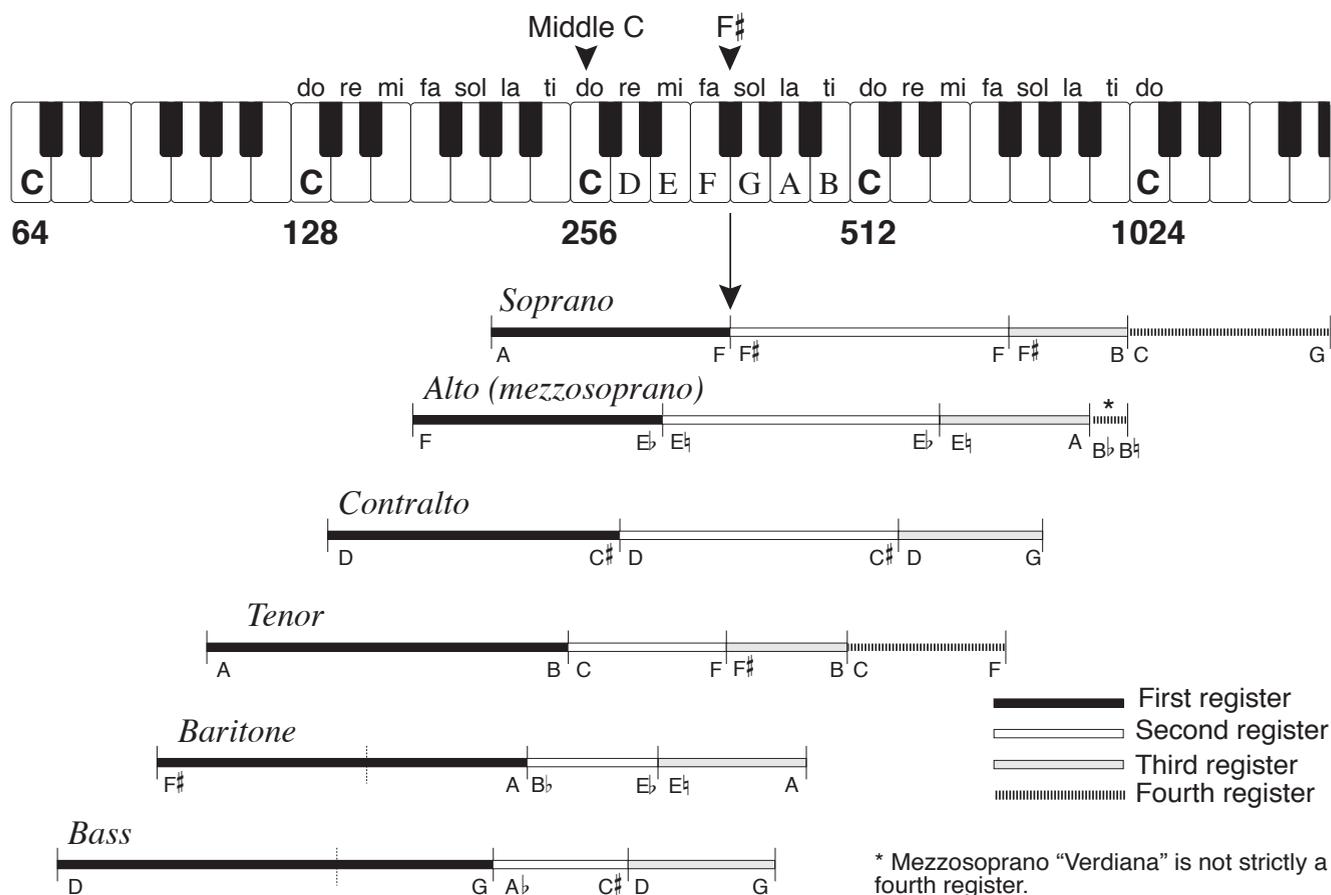
For this writer, any effort dedicated to the defense and enrichment of the practice of classical fine arts, classical music most emphatically, is always a labor of love. That is the principal motivating purpose here; however, this report and the shaping of its content as a

whole, are prompted by three additional considerations.

1) Modern Western European civilization, including that of North and South America equally, rests upon the reaffirmation and further development of Augustinian principles accomplished by chiefly the fifteenth-century Italian Renaissance, as centered around the work of the 1439 Council of Florence. Modern classical fine art is an integral part of the continued functioning of that heritage.

Presently, the very existence of that civilization is imperilled by cultural warfare against everything associated with the Augustinian heritage generally, and the heritage of the Council of Florence most emphatically. The most visible expression of this cultural warfare imperilling our civilization is everything typified by bolshevism and the satanic influences centered around the

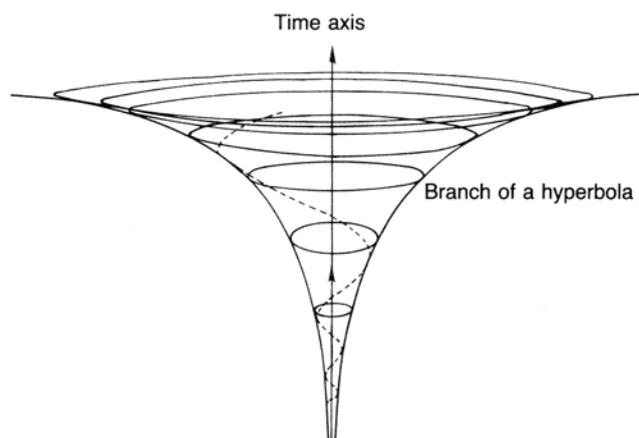
FIGURE 3
The Six Species of the Human Singing-Voice



Source: Schiller Institute research team. Ranges are based on known examples in the classical vocal repertoire.

FIGURE 4

Model of an Ordinary Hyperbola



The surface of revolution of one branch of the hyperbola is the least-action representation of the process of the flaring horn. This mapping of the singularities of a Riemannian hyperspherical continuum is correct, but ultimately inadequate.

work of Theodore Adorno’s Frankfurt School, and the satanic evil of the Adorno-Arendt dogma of “the authoritarian personality,” the cultural dogma of the modern Anti-Christ.

Under such circumstances, the defense of classical music against both the Muscovite influence and the quite literally satanic irrationalism of such as Adorno, is an indispensable feature of the defense of Western civilization as a whole.

This defense requires that principles of music be liberated from the authority of sundry musicological and related cults of “art for art’s sake,” including the gifted Shenker’s false, Helmholtzian dogmas on tuning and irrationalist absurdity of “absolute music.” The essence of classical fine art is, as we have indicated for classical music, not a matter of varying mere musicological opinions; classical composition expresses a principle which is implicitly a fully intelligible one, and subject to the same authority of proof as a theorem in physics.

2) The intelligible representation of a quadruply-connected manifold, the minimal conception of classical composition, is, at a minimum a proposition in topology lying specifically within the domain of Riemannian physics. However, this representation is not, by itself, an adequate one.

The functional mapping of the singularities of a Ri-

emannian hyperspherical continuum, is merely approximated by the relevant point-set mapping of hyperbolic-trigonometric forms of discontinuities. That mapping is correct, as far as it goes, but is ultimately an inadequate one. (Cf. Figure 4—model of an ordinary hyperbola.)

In brief, on this specific point, self-evident points do not exist in the non-euclidean constructive geometry upon which the Gauss-Riemann complex domain rests. Therefore, the existence of such determined points, as points, has no causal significance in a continuing, non-linear process, but the purely negative one, discontinuity, addressed by Dirichlet. Had the Riemann Surface Function been examined more thoroughly than it has been, this problematic feature of that function would have received wider attention than it enjoyed in the hands of Riemann’s student, collaborator and critic, Eugenio Beltrami.

This problem has systematic relevance for thorough comprehension of the quadruply-connected, minimal domain of classical musical composition. It has crucial relevance for leading questions of experimental physics today.

The problem is, to restate the nature of the Riemannian point-set, both with respect to each point and to the topology of the Riemann Surface as a whole, to the effect of eliminating the paradoxical nature of the point as such. The solution of that paradox accords, at first and second impressions, with Beltrami’s arguments on the subject of negative curvature. Beltrami’s argument accords also with the writer’s geometrical definition of *negentropy*, as adduced from a refutation of a related problem, his refutation of the axiomatic fallacy underlying the so-called Kantian Paradox. This bears also upon the solution to the Parmenides Paradox, whose form of solution is the central feature of Nicolaus of Cusa’s work founding modern non-euclidean geometry, *De Docta Ignorantia* [On Learned Ignorance]. The writer’s own formal solution to the Parmenides Paradox was elaborated as a feature of his refutation of Kant.

The writer has identified some of a set of crucial experimental problems of present-day physics, which beg implicitly the issues of Beltramanian negative curvature, and has proposed that the history of the development of the conceptual basis of modern physics be traced from relevant work of Brunelleschi, to the present, to the purpose of putting the conceptual nature of the problems in appropriate historical focus.

The history of the refinement of the principles of classical polyphony, since the period of Brunelleschi,

through the work of Kepler and beyond, is intertwined with the developments leading into the articulation of Beltrami negative curvature. Otherwise as indicated proximately above, there are conceptions bearing upon the principles of classical composition which beg attention to the same conceptual matters.

3) The memoranda which this writer has circulated earlier to sundry scientists and other researchers, on the subject of an historical background approach to the issues of Beltrami negative curvature, have already borne useful fruit, including a report on relevant work-to-date among the writer's associates, including a significant such report by Dino de Paoli.

Elements of that latter report help to situate the issues of quadruple-connectedness. I shall quote passages from that report, and reference some of the illustrations supplied.

A surface of constant negative curvature, or the so-called Beltrami pseudosphere, has the topological characteristic of triple connectivity. There are at least two singularities built into the surface, which does not allow a simple topological closure. In contrast, a surface of constant positive curvature, or the so-called Riemannian sphere, has the topology of a simply connected surface, with only two poles instead of singularities. (See **Figure 5**.)

Beltrami himself could not find a full physical meaning for such a surface, except the obvious one of a surface of minimum action.

The direction you indicate seems to me absolutely the right one. I had an unfinished idea on the use of the Beltrami Surface as representing a potential surface characterizing what are indicated today as *gravitational* and *strong* (nuclear) interaction, for reasons I have to elaborate fully.

The positively curved surface, represents the potential surface of the so-called *weak* and *electromagnetic* interaction, which it is known are coupled. The unification of all interactions in the direction of a *continuous* surface, where the positive/negative curvatures are not simple polars, but where indeed the negative represents "holes," but of the Beltrami type.

"The dissymmetry in the negative/positive

ratio allows the definition of absolute time, and the consequent non-respect [non-correspondence] of the conservation of energy in a single manifold (the neutrino problem, for example).

He then discusses some simpler kinds of crucial-experimental demonstrations. He turns first to the geometry of the generation of the shock-wave, in Riemann's 1859 *On The Propagation of Plane Air Waves of Finite Magnitude*. Referencing an example developed in his published article on Leonardo da Vinci,

Take the surface of water as representing an equipotential surface. The energy of the surface tension is then seen geometrically. A simple sine-wave, including a soliton, is characterized and mathematically representable by a surface of positive curvature (elliptic function). (See **Figure 6**.)

The formation of water breakers—that is, the "breaking of the surface"—creates a topological transformation, given that the breaks, or holes, or singularities increase the connectivity, and, mathematically, with shift to hyperbolic functions (Beltrami). That is, a surface of negative curvature. This is the shock-wave description [of this phenomenon, according to the referenced paper of Riemann].

Referencing his papers published earlier, he supplies a second simple kind of crucial illustration. His reference to the glass of wine should signify a wine-glass whose bowl is of constant positive curvature, and red wine produces the clearest results for laymen repeating this demonstration.

If you take a glass of wine, and put it in front of a lamp, the projection of light on the table does not produce a *point-light*, but a characteristic figure called a *caustic*. (**Figure 7**.)

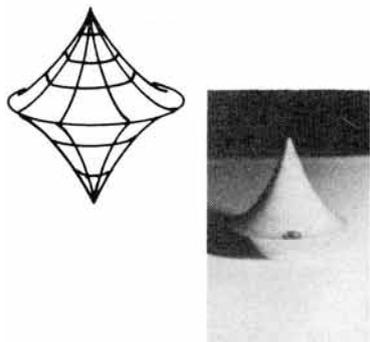
If you try to transform perspective linearly on a curved surface, a simplistic interpretation of the so-called *Leonardo* curved perspective, you end up with the following problem. Your *focal points* are no longer points, but *caustics*. (**Figure 8**.)

He concludes with the following summary:

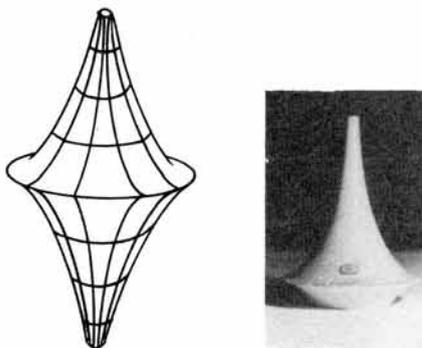
FIGURE 5

A) Surfaces of constant negative curvature elaborated by Beltrami

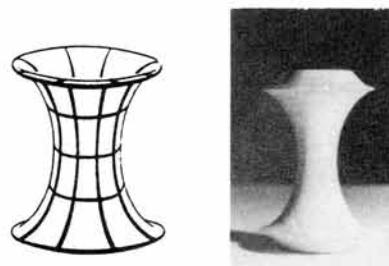
A-1 Solid generated by the rotation of a caustic



A-2 Pseudosphere generated by a tractrix (see Figure 5C)



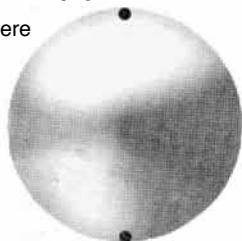
A-3 Catenoid type generated by rotation of a catenary or a cycloid (see Figure 5C)



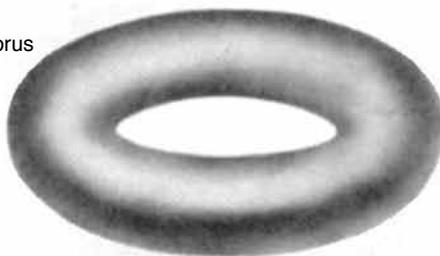
Beltrami showed that there are only three constructible solids of constant negative curvature. He named only one of these, the pseudosphere. The photos by Dino de Paoli show Eugenio Beltrami's original models, which are kept at the University of Pavia in Italy.

B) Multiply connected surfaces

Sphere



Torus



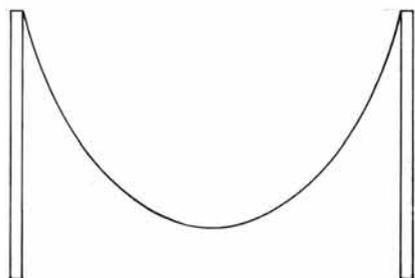
Pretzel



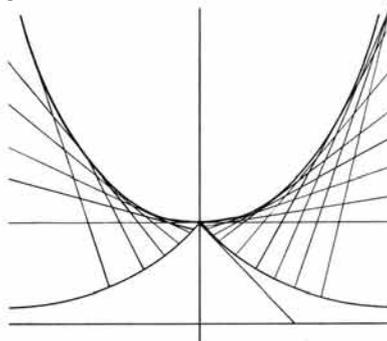
The topology of the projection of a sphere (constant positive curvature) has simple connectivity; there are no singularities (holes), only poles. The projection of a torus, with its center hole, is doubly connected, and the projection of a pretzel shape, with two holes, is triply connected.

C) The catenary and the tractrix

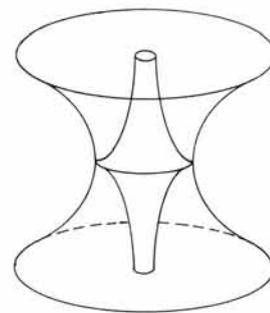
C-1



C-2



D) Combination of two negative surfaces generated by the catenary and tractrix



The catenary is the form assumed by a chain or rope suspended from two fixed points and hanging under its own weight (C-1). To find the involute of a catenary (or of any curve), imagine a thread on the surface of the curve, which is then cut and unwound from the lowest point on the curve to the left and right. The ends of the thread on a catenary rope trace out the tractrix shown below. Each step of the unwinding is like constructing a tangent of the catenary to the tractrix. If the normal (perpendicular) is drawn to the tangent of the tractrix at any point, it can be seen that this normal becomes a tangent to the catenary. Note that all tangents from the inside of the tractrix to its base are equal in length.

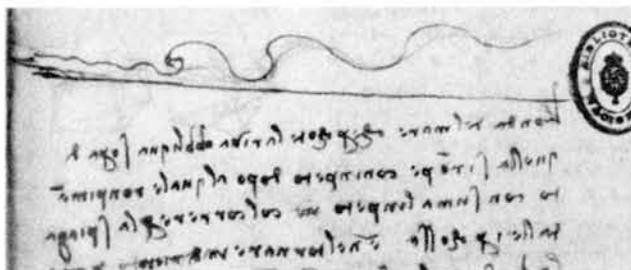
Of this type of curve, called "mechanical curve," the most general is the cycloid. The cycloid is important in two respects: It has the physical characteristic of being the path which requires the least time for a body to descend from one point to another (brachistochrone). Also, descending bodies all arrive at the end of the curve at the same time, independently of the initial position (isochrone).

FIGURE 6

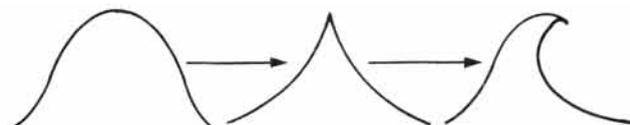
A) Simple Sine-Wave with Underlying Parabolic Geometry



C) Leonardo's drawing of wave with breakers forming



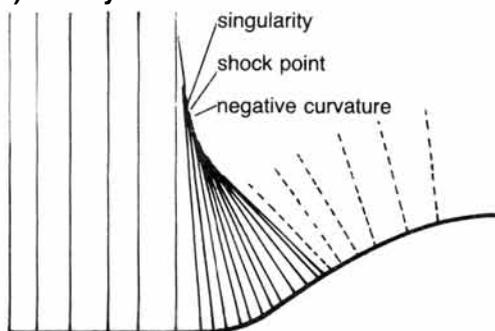
B) Formation of a Breaker: Schematic



D) Breaker with surf-rider



E) Theory of characteristics



What Riemann called “geometric characteristics” and Leonardo called “cross waves,” are represented by perpendicular lines when the speed is constant, and bend right or left when the speed increases or decreases, for example due to enlarging or narrowing the passage through which a fluid is flowing. Thus it will appear that the characteristics touch. Riemann used this to represent a shock wave. It is also a singularity. It is also, clearly, negative curvature, which therefore appears in connection with the formation of a singularity.

The amount of Leonardo’s illustration on this field, and on the related reflection on a curved surface, proves that he faced it, and solved it.

His solution, through the discovery of the parabolic or elliptic mirrors, which eliminate the caustic, and reestablish a *focal point*, is geometrically equivalent, in avoiding of projective distortion between a curved surface and a plane, to using what is called “Gaussian” curvature, that is, the elliptic type. (Figure 9.)

But, projectively, Leonardo’s solutions, through the compass of proportion, leads, if elaborated, to Desargues’ theorem, and, more interestingly, to the establishing of proof of the projective invariance of the Golden Section.

As said, such *caustics* are second-order Beltrami surfaces; they are key. Briefly, they were then studied geometrically by Kepler, [Chris-

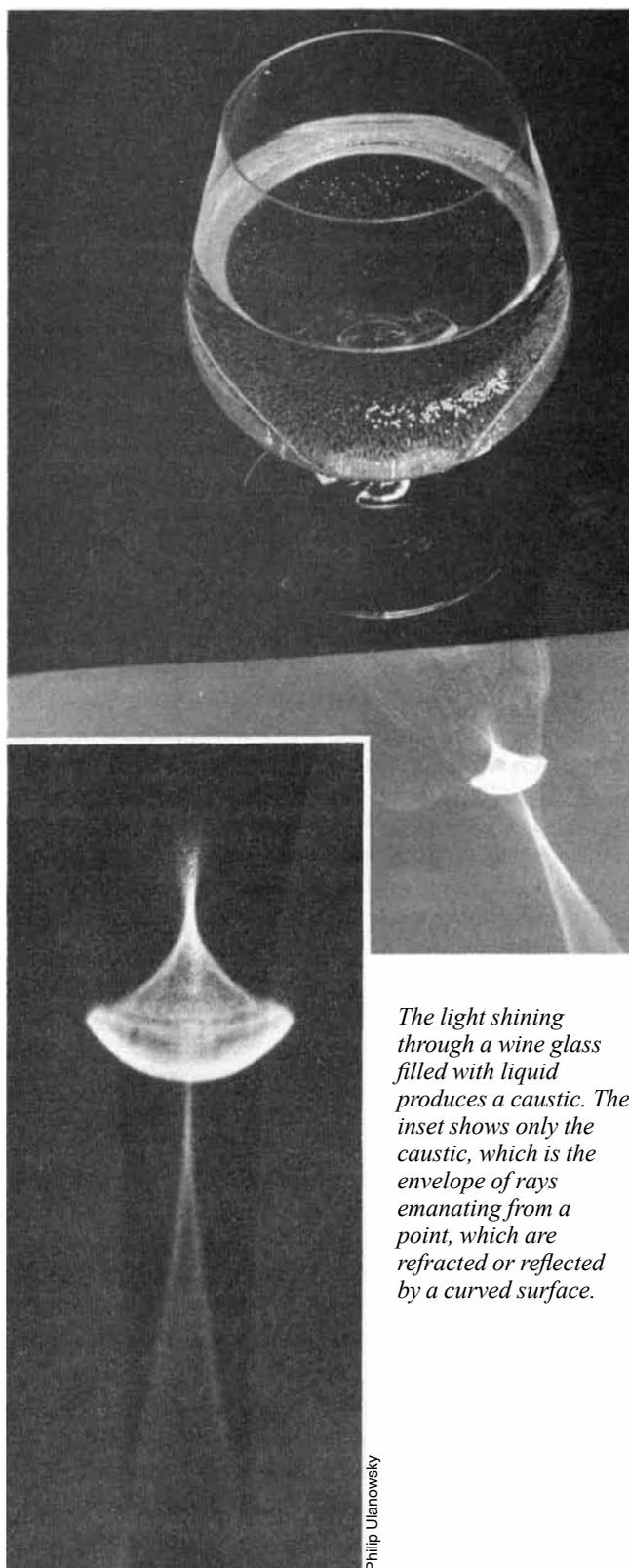
tiaan] Huyghens, but especially by [Gaspard] Monge. (Figure 10.)

A step further, and the same method, at least visually, leads to the graphic use of the *characteristic lines* by Riemann to describe the shock-wave [On The Propagation of . . . , 1859].

As we introduced this material here, the foregoing quotation, and accompanying figures supplied by de Paoli, were one of numerous responses to an earlier research memorandum. The issuance of that memorandum has a pre-history, which is relevant to the purpose of citing de Paoli’s remarks and illustrations in connection with the quadruply-connected geometry of the classical musical domain.

For more than two decades, this writer had been persuaded that the physical space-time curvature of sub-atomic microspace must be necessarily represented as a Riemannian space of harmonically ordered

FIGURE 7
Caustics



curvature. This view had been reworked into an hypothesis at the beginning of the 1970s. At that time, the practical function assigned to that hypothesis, was the mapping of the characteristic functions of development of the biosphere, on the working assumption derived from the hypothesis, that the primary location of the *negentropy* characteristic of both living processes and their interaction with inorganic processes, is located in the sub-atomic domain of microphysics, and that the physical geometry of that domain is itself representable as *negentropic* in Riemannian geometric terms of reference.

This writer had “shopped” the hypothesis into a number of the scientific seminars in which he participated. In 1985 and 1986, that hypothesis was taken up, and a crucial-experimental proof of it supplied. The evidence is now conclusive, that hypothesis is the correct one. On the basis of the submission and discussion of that proof, a number of new projects were set into motion, including a fresh approach to the determination of the Periodic Table of elements and isotopes.

Several of those lines of work, but the investigation of the Periodic Table’s ordering of the protons and neutrons of the nucleus most emphatically, posed the importance of defining in a Riemannian way the *strong* nuclear forces. It became clear to this writer, that the referenced inadequacies in elaboration of the Riemann Surface Function were put at the center of the crucial-experimental work needed to address the proposed nuclear ordering.

For several reasons, Beltrami’s treatment of negative curvature appeared to be a profitable line of approach. Some useful work by a young Italian researcher, completed a few weeks ago, prompted me to set this line of approach into high gear. What was needed to that purpose, was to define a multi-faceted, but coherent approach to several lines of investigation. In such a case, this writer resorts to a tactic which one senior physicist has described as “provocative hypothesis.” A provocative memorandum was written and circulated, intended to provoke as many fruitful, parallel but converging lines of reflection and inquiry into motion as possible.

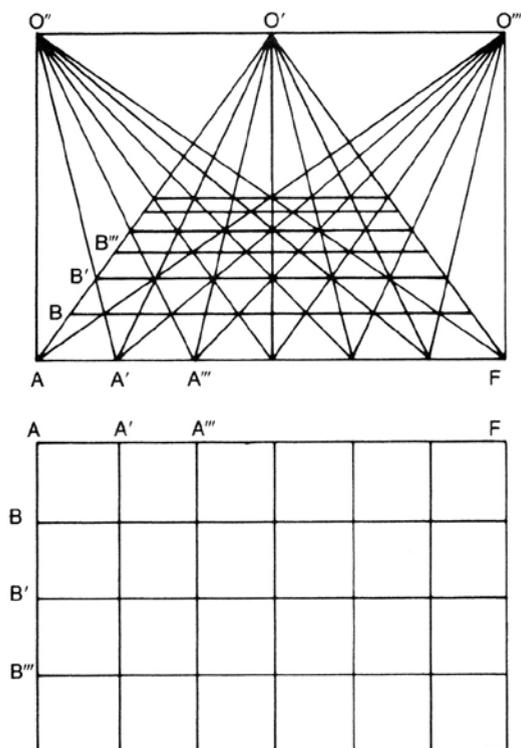
That was one of two memoranda on this matter, to which researcher Dino de Paoli responded in the manner indicated above.

Some of the considerations involved in that research memorandum bear directly on the relevance of this to the principles of musical composition. Those considerations are identified next.

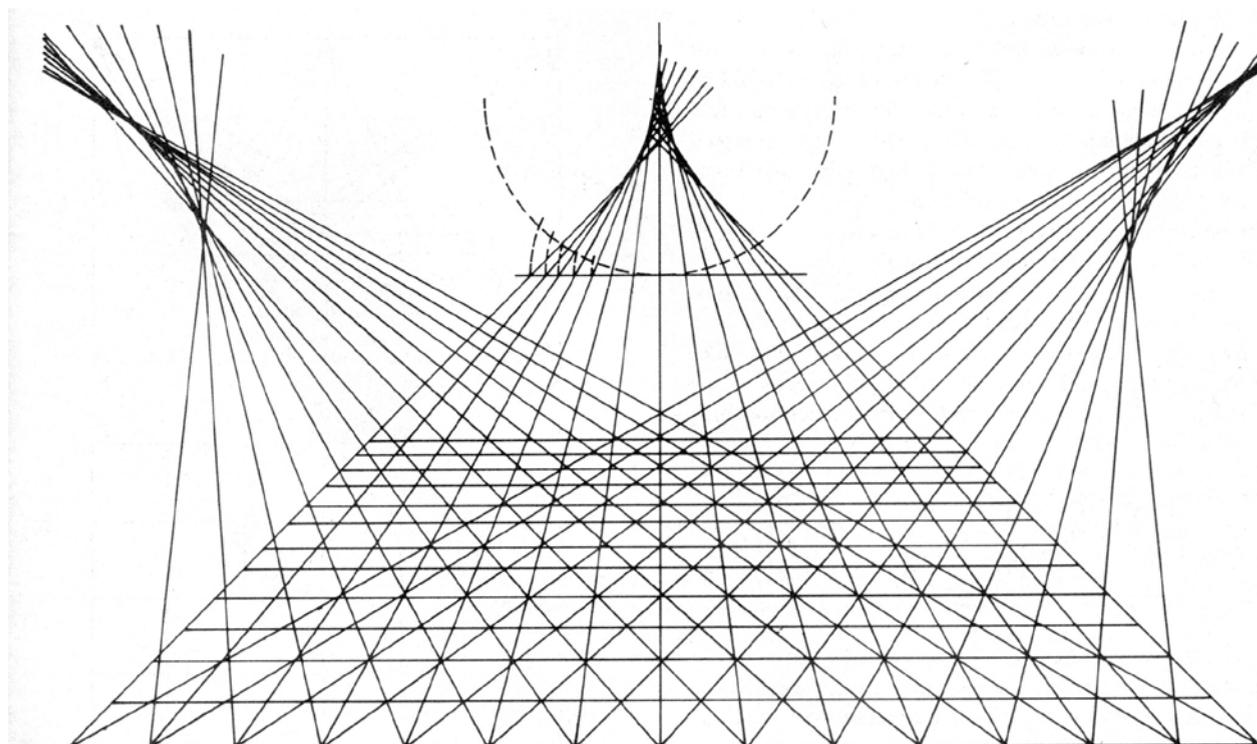
FIGURE 8

Perspective constructions

A) Simple linear perspective



B) A simple reflection between linear perspective and a curved surface produces caustic points instead of focal points.



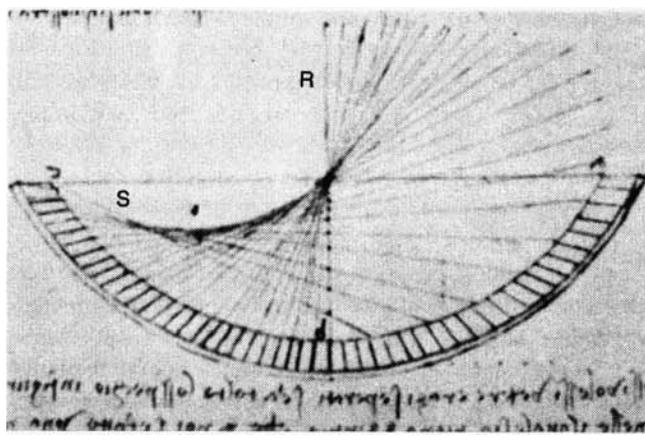
Implications of the Kantian Paradox

Whenever we are confronted with crucial evidence requiring us to overturn hallowed presumptions of prevailing scientific opinion, we must take two approaches simultaneously. We must direct our attention to relevant sorts of contemporary scientific work, but we must also examine both the conception we are overturning, and its proposed successor, from the standpoint of relevant features of the internal history of science.

We must examine the underlying assumptions of extant prevailing scientific opinion, in their character as assumedly axiomatic assumptions of method and ontology, and do this in a Socratic way. We must, simultaneously, examine the kindred roots of the kinds of notions of method and ontology we are putting forward as alternatives, variously explicitly and implicitly. We must also combine these two approaches to the relevant internal history of science, to trace the history of interaction between the tendencies toward, and conflicts among the contrasting views under consideration.

Since, in all such cases, we are addressing directly “axiomatic” qualities of conception of method and ontology, our attention is focussed almost entirely on that which is profound and simple, and to the crucial experimental evidence which always corresponds to the most profound, and hence simplest matters.

FIGURE 9
One of Leonardo da Vinci's Drawings of a Caustic



RS represents the caustic.

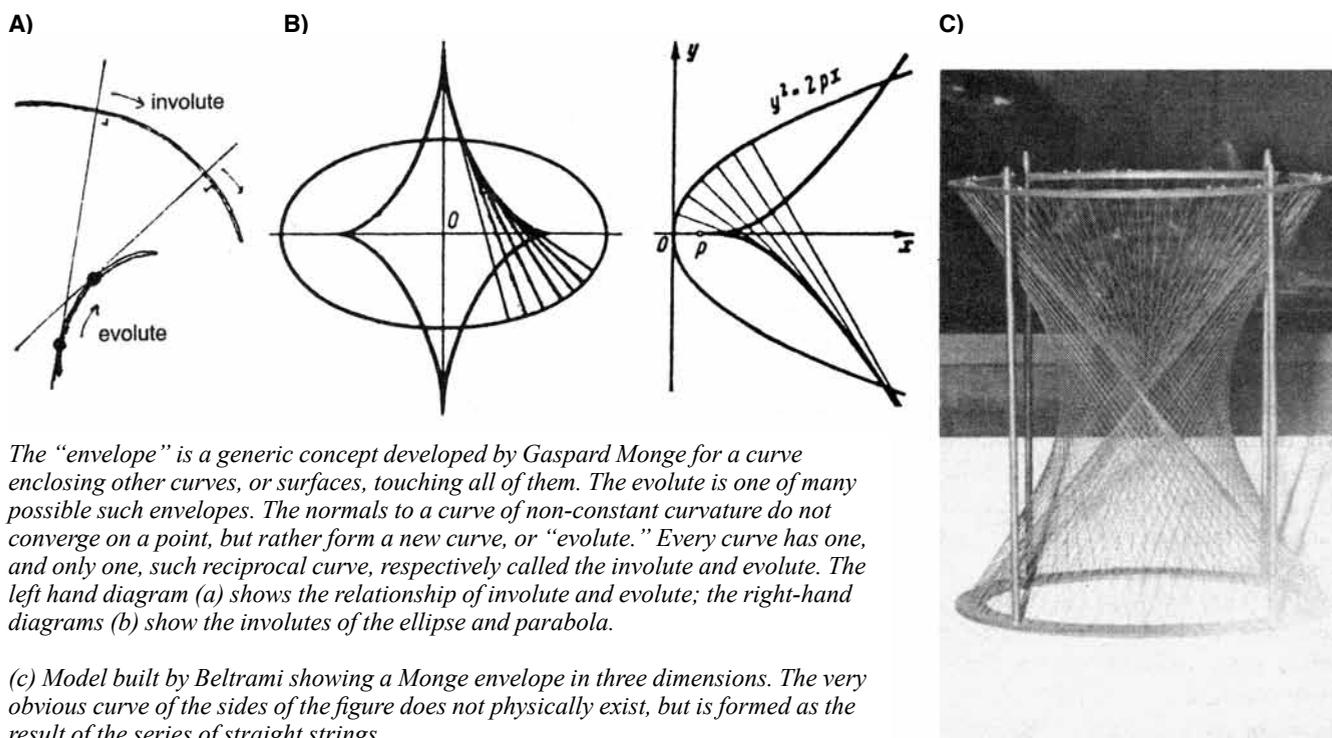
It is the case, that all of the fundamental laws of physics, and crucial evidence pertaining to fundamental laws, are always properly to be examined in terms analogous to the approach of Kepler. For reasons of the physics of non-euclidean geometries, fundamental physical laws are always rooted in congruence with a

definite curvature of physical space-time. Indeed, any statement respecting the curvature of physical space-time, and any statement respecting elementary notions of universal physical laws are, respectively, but two ways of saying the same thing, as the instance of the *fine-structure constant* merely illustrates the point in an implicitly conspicuous way.

Contrary to the popularized delusions associated with a formal euclidean, deductive geometry, such as a Cartesian form of Newtonian or formal-statistical discrete manifold, all physical space-time is curved in effect. This curvature reflects a self-bounded character of physical space-time as a whole. This has two practical implications which must be drawn out here.

First, as Kepler's astrophysics illustrates this, the fundamental laws of motion in physics are not determined by the interaction, as "at a distance," between two bodies in empty space and empty time. The fundamental laws of motion exist independently of any bodies affected by them, and independent of any ideas commonly associated with a Cartesian or Neo-Cartesian sort of discrete manifold. Those laws exist everywhere in the universe, at all times, in a manner independent of any consideration associated with ideas analogous to a constant speed of light.

FIGURE 10
The Monge Envelope



The "envelope" is a generic concept developed by Gaspard Monge for a curve enclosing other curves, or surfaces, touching all of them. The evolute is one of many possible such envelopes. The normals to a curve of non-constant curvature do not converge on a point, but rather form a new curve, or "evolute." Every curve has one, and only one, such reciprocal curve, respectively called the involute and evolute. The left hand diagram (a) shows the relationship of involute and evolute; the right-hand diagrams (b) show the involutes of the ellipse and parabola.

(c) Model built by Beltrami showing a Monge envelope in three dimensions. The very obvious curve of the sides of the figure does not physically exist, but is formed as the result of the series of straight strings.

In effect, every two-body problem is a three-body problem, in the sense that it is the interaction of each of two discrete bodies with the curvature of space-time, which determines their apparent motion with respect to one another.

Since Gauss, we have understood more clearly than before, that the adducing of the most elementary laws of physics depends upon discovering means by which we, with the limitations of our perceptual apparatus, might map the curvature of the physical space-time in which we exist. Implicitly, since the work of Nicolaus of Cusa, and the continuation of Cusa's work by Leonardo da Vinci and his associates, we have understood that there are three special domains of experience in which this mapping must be conducted. These three domains represent, geometrically, the extremes corresponding to the self-bounding of physical space-time as a whole. These three are, the extreme scale of astrophysics, the extreme scale of microphysics, such as sub-atomic microphysics, and the characteristics of all living processes which characterize the difference between living and non-living processes on the ordinary scale of perceptual experience.

No proposed law of the universe can be considered a law unless, and until it has been demonstrated to be characteristic commonly of all three boundary conditions: astrophysics, microphysics, and optical biophysics.

Whenever we examine the internal history of science respecting elementary issues of method and ontology, our retrospective view must be concentrated upon topics which either were, or ought to have been premised simultaneously on evidence bearing upon astrophysics, microphysics, and optical biophysics as domains in which crucial evidence is sought. On this account, the figures Nicolaus of Cusa, Leonardo da Vinci, and Kepler, are paradigmatic for strict usage of the term "scientist."

My own discoveries, bearing upon fundamental principles of economic science, have rendered intelligible a fourth boundary condition, the characteristic curvature



Immanuel Kant (1724-1804) claimed that creative processes for valid fundamental scientific discoveries are beyond all human understanding. LaRouche proved him wrong.

of those kinds of mental processes associated with generation of a valid fundamental discovery in science, processes otherwise exhibited in the instance a student, for example, effectively replicates, in his or her own mind, the processes by which a valid fundamental discovery was originally constructed.

So, on the latter account, true education never bases itself upon a student's drill and grill in the mastery of isolated "facts." Rather, true education, as in the footsteps of Groote's Brothers of the Common Life, or the Humboldt reforms, devotes the greatest portion of the teacher's and student's attention to the student's reliving the mental experience of generating a past discovery. De-

fective education may produce students matriculating with high marks, but they are for the most part merely victims of a process not very much dissimilar from the programming of a moronic digital computer, to elevate it to the status of an "idiot savant."

So, today, we have graduates, even many with terminal degrees, graduating with highest marks, who are incurably bunglers whenever the assigned task requires real thinking. Such is the included effect, of the drift in the direction implied by "programmed learning."

My most important discoveries, in every field to which I have contributed, are based upon my successful refutation of the famous Kantian Paradox reasserted in Immanuel Kant's *Critique of Judgment*. Kant asserted two things of relevance here.

First, he insisted that although creative processes responsible for valid fundamental scientific discoveries exist, these processes themselves are beyond all possible human understanding. That I proved to be false, and from that proof developed an approach to intelligible representation of those creative processes, and hence the implicit measurement of technological progress as such.

Second, on the basis of the first assumption, Kant argued that there were no intelligible criteria of truth or beauty in aesthetics. The toleration which has been gained so generally by all modern irrationalism in mat-

ters of art, has depended upon German and other acceptance of this thesis on aesthetics advanced by Kant and Carl Savigny later.

On condition that we show, that classical fine art depends upon the generating function of the same individual creative mental processes otherwise responsible for the generation and assimilation of valid fundamental scientific discoveries, and only on condition of that proof, are we able to supply valid general statements about “human nature.”

It happens to be the case, that the ordering of the creative mental processes is characteristically *negentropic*, not in the way absurd, popularized “information theory” employs Ludwig Boltzmann’s H-theorem, but in the way in which Leonardo’s and Kepler’s definition of the harmonic ordering of living processes is apprehended from the standpoint of the Gauss-Riemann constructive-geometric mapping of the complex domain.

In effect, the function of creative-mental processes to generate conscious, willful states of the human mind, as no animal species-member can do, and the further possibility of rendering these creative processes themselves conscious, by means of rendering them intelligible, represents man as a living process in whom the characteristic *negentropy* of living processes generally has become willful consciousness. In man, life is enabled to act upon itself by “free will,” and upon the universe as a whole, too.

Since all formal knowledge of science and other matters is only relative, subject to future scientific revolutions, the question is posed implicitly to us: “How can we pretend to know anything?” The answer is, in terms of those kinds of thought we associate with simple irrationalism, or even with methods of deductive formalism, we know nothing with certainty, and are usually in more or less grave degree of error in our opinions. How, then, can we ascribe the authority of even relative certainty to science?

What we can demonstrate, is the increased per-capita power of mankind over nature as a whole through those processes subsumed by the term “technological progress.” What we can show, in this way, is that “technological progress” to such effect is *truth*. This fact locates true knowledge uniquely in the relevant functioning of the creative mental processes, by means of which fundamental scientific progress is generated and assimilated. Formal, deductive statements are relatively true, only to the extent they borrow a shadowy authority from the functioning, not of formal-deductive processes, but of creative processes.

All human scientific, and artistic knowledge is so premised as relatively true knowledge. Thus, everything we might presume to know respecting the curvature of physical space-time depends ultimately upon this scrutiny of the mental processes by which knowledge is developed. Thus, self-consciousness of an implicitly intelligible representation of the individual’s creative-mental processes, is a bounding condition, and implied test, to which all scientific opinion must be subjected. Hence, this, added to crucial features of astrophysics, microphysics, and optical biophysics, is the fourth bounding condition of all human knowledge respecting the elementary laws of the universe.

For reasons so implied, this view is the only vantage-point from which the essence of classical musical composition can be adduced.

Now, to the Point

This brings us to the juncture, at which the importance of Beltrami’s work is shown, both in a general way, and then its bearing upon the principles of classical musical composition. Earlier, we have indicated that we can not be misled into treating the points of a Riemannian point-set as if they were “points” in the same sense euclidean deductive geometry defines points axiomatically. In music, the introduction of a *relative* harmonic or metrical dissonance occurs as the generation of a point in a Riemann Surface; hence, the general case and the musical case are conjoined.

For those readers unfamiliar with the author’s proof, it is essential that we supply a summary of the most relevant features here.

To the degree mathematical-physics formalism adopts the standard of consistency associated with deductive method, the entirety of such a physics forms what is termed a theorem-lattice of the form analogous to a euclidean geometry premised upon an underlying set of arbitrary (i.e., unproven) assumptions classed as a set of axioms and postulates. In such a theorem-lattice, there exists no single consistent theorem which states anything which is not already implicitly asserted by the corresponding set of axioms and postulates. This connection is frequently referenced as “the hereditary principle.”

Therefore, in the instance a crucial experiment demonstrates any consistent theorem of such a theorem-lattice to be false, this suffices, by virtue of the “hereditary principle,” to prove that the underlying set of axioms and postulates contains something false. It therefore also demonstrates that the entire theorem-lattice of mathematical physics is permeated by a corresponding,



Mr. and Mrs. Lyndon LaRouche (at concert, lower right) have actively sponsored a musical renaissance. Other photos, clockwise from upper left: Baritone Piero Cappuccilli shows the difference in a Verdi aria sung at the scientific pitch of $A = 432$ and the modern, higher tuning, at a 1988 conference held by Helga Zepp-LaRouche's Schiller Institute in Milan; French 'cellist Eliane Magnan has recorded all the Bach solo suites at Mr. LaRouche's encouragement; violinist Norbert Brainin (former first violinist, Amadeus Quartet) and pianist Günter Ludwig at a Washington, D.C. benefit for Lyndon LaRouche's legal defense in December 1988.

“hereditary” axiomatic falsehood.

Let a theorem-lattice so discredited “hereditarily” be designated for reference as Lattice A. Any alteration of the set of axioms and postulates effected to correct the errors shown to exist in one theorem, as shown by a crucial experiment, defines therefore a second theorem-lattice, “hereditarily” consistent with the reformed, new set of axioms and postulates, the which we may designate for reference as Lattice B.

It may be the case, and often is, that it is suggested that several alternate modifications of Lattice A's existing set of axioms and postulates, might appear to satisfy the requirement of altering the crucial theorem in such a way as to appear to agree with the crucial experimental findings. Thus, we would have, in such a case, an implicit series of mutually-exclusive choices of theorem-lattices, B, C, D, and so on.

The question, which of these alternate lattices is the proper choice, is accomplished by treating every theorem in each such hypothetical lattice as if were implicitly a new crucial experiment. We require a new lattice

which is not only consistent, but in which each derived theorem is in agreement with the relevant crucial experiments implicitly defined by that theorem. The completion of such a process, set into motion by a single crucial experimental disproof of one theorem of an existing mathematical-physics theorem-lattice, is the formal meaning of the term “scientific revolution.”

Let us presume that the result of such a scientific revolution is Lattice B. Let us examine the result under two conditions: firstly, as a general condition, for all such cases; and, secondly, under a very specific, idealized condition.

In the general condition, by virtue of the “hereditary principle,” no theorem of Lattice B is consistent with any theorem of Lattice A, and vice versa. Thus, there exists such a “logical gap,” an unbridgable gulf, in fact, separating any possible theorem of Lattice B from any possible theorem of Lattice A. In formal, deductive mathematics, the name for such a “logical gap,” is a *mathematical discontinuity*. In physics, the name for such a phenomenon, is a *physical singularity*. The

notion of a “topological singularity,” as referenced by de Paoli in his cited remarks, has the same significance.

In the ideal case, let us assume that the crucial experiment appears to require only the smallest possible degree of change in the underlying set of axioms and postulates. A change in the parallel postulate of euclidean geometry, as already implicit in Desargues’ theorem, is an example of such an ideal case. Examine the logical gap, mathematical discontinuity, or singularity, generated between euclidean Lattice A and neo-euclidean Lattice B, by “hereditary” implication.

Since the “logical gap” so defined between the two respective theorem-lattices is of the smallest degree possible, there exists no alternate theorem-lattice, alternate with respect to either Lattice A or Lattice B, which could make the resulting gap between the two lattices deductively intelligible.

That preliminary conclusion, reached by that route, subsumes what Kant mistook for a conclusive proof that the creative processes are not susceptible of intelligible representation for the human understanding.

It should be clear, without more explanation than merely mentioning that fact, that Kant’s argument depends upon the presumption, like Descartes’ and Newton’s identical error before him, that his notion of a neo-Aristotelean deductive method is the purest form of human reason.

There is no innocence in Kant’s assertion of this. The entirety of Kant’s work, both before and after his distancing himself from Hume’s “philosophically indifferent” turn to a radical form of empiricism, is devoted to the single purpose of seeking to extinguish even the memory of the work of Gottfried Leibniz from German thought. The central issue in this undertaking of Kant’s, is the fact that all of Leibniz’s work in science and statecraft depends upon a view of the implicit intelligibility of the creative process, to which Leibniz refers in such locations as his *Monadology*.

This issue brings us to a deeper problem of method and ontology than that explicitly represented by the problematics of deductive theorem-lattices. If crucial experimental evidence demonstrates the kind of discontinuity shown as a logical gap between two deductive theorem-lattices, to correspond functionally to the existence of a physical state, then, in that case, the crucial experiment suffices to demonstrate that all deductive method is premised upon pervasive methodological and ontological absurdity.

In that case, rather than correcting the set of axioms and postulates of Lattice A, we throw all of them out,

and deductive method and correlated ontological assumptions with them.

At this juncture, it is important to stress, for those readers unfamiliar with this distinction, that our remarks above referenced a change in the parallel postulate as generating not a “non-euclidean geometry,” but rather a “neo-euclidean” one. This distinction bears in a crucial manner and degree upon the popularized error of representing a change in the parallel postulate as generating a “non-euclidean geometry,” when this merely produces a “neo-euclidean” one. This is key to the pervasive nonsense to this effect, so widely advertised in efforts to represent Special and General Relativity; it is key to the formal reasons why the learned disputes upon the subject, whether Riemann curvature of physical space-time must incorporate the negative curvature of Beltrami, are such useless muddles.

By a “non-euclidean geometry,” we signify a purely constructive geometry, which prohibits any set of deductive axioms and postulates, and prohibits any employment of the deductive method in the elaboration of the theorems of geometry, or of mathematical physics in general. The modern discoverer of “non-euclidean geometry” was Nicolaus of Cusa.

Narrowly, as he reports this fact in some among his sermons, Cusa discovered what modern textbooks identify as the *isoperimetric theorem* of topology, as a solution to the problem which Archimedes had treated in the latter’s theorems addressing the subject of attempts to square the circle. In his *De Docta Ignorantia*, Cusa situates the results of that proof in a general form of solution for what is usually known as the Parmenides Paradox.

In a more limited respect, as this bears upon the matters immediately under consideration here, is the following.

The only consideration from which a non-euclidean geometry begins, is that the intelligibility of developments in this universe must be constructed by reference to nothing but the relative maximal result effected by the relative minimal action. This is the root of the famous central principle of physical science, as first rigorously defined by Leibniz: the universality of a principle of physical least action. This is Cusa’s “Maximum Minimum” principle.

In the simplest case, this yields the isoperimetric theorem. What is the minimal perimeter encompassing the relatively largest area or volume? This proof defines the circle or sphere in a socratic way, to such effect that the proof is independent of any consideration employed

in demonstrating it. The method of this proof is the nature of what Leibniz termed *analysis situs*, later termed *topology*. (There are different, defective guises of taught topology, but we may ignore them here.)

From this beginning, a *constructive* or *synthetic* geometry, otherwise the strict definition of a *non-euclidean geometry*, is elaborated. This is the basis for construction of Riemannian and Beltramanian geometry, and thus the key referent for the propositions considered here. The paramount considerations here, are three:



Gottfried Wilhelm Leibniz (1646-1716), like Leonardo da Vinci and Kepler before him, premised his physics upon a non-euclidean geometry of the type described here.

- 1) That circular action is the root-notion from which the notion of physical least action is derived;
- 2) That circular action is the only standard of measure in physics;
- 3) That, to construct a geometry, we can not begin with less than doubly-connected circular action, and preferably triply-connected.

By “doubly-connected circular action,” we signify that every circular action is acted upon, in every smallest imaginable interval, by a second circular action, upon which it acts, similarly, in turn. In “triply-connected circular action,” a third circular action acts similarly upon, and is acted similarly upon, each of the two of doubly-connected circular action.

Such *multiply-connected circular action* suffices to generate points and so-called straight lines. Hence, at this instant, points cease to have any self-evident existence, since we have shown that they have a fully intelligible existence, as generated by construction. The same applies to the generation of so-called straight lines.

From this beginning, the entire scope of the theorems of plane and solid euclidean geometry is generated, solely by construction, never considering anything not generated by nothing more than *multiply-connected circular action*. Hence, all sets of deductive axioms and postulates are outlawed from geometry, and mathematics generally, and the deductive method, too. That is a *non-euclidean geometry*.

The physics of Leonardo da Vinci and Kepler is premised upon such a non-euclidean geometry, as is the physics of Leibniz after them.

Without nullifying anything in such a multiply-connected circular-action geometry, Gauss, Dirichlet, Riemann et al. produced a superseding form of *synthetic geometry*, upon which the work of Beltrami is also premised. In this higher *synthetic geometry*, we express circular action in the conic form of self-similar-spiral action: denoting, that in our universe, physical least action is expressed by a constant, self-similar increase, or decrease of the magnitude subtended by circular action.

In a strict application of multiply-connected circular action to physics, the extension of circular action in time is represented by a cylinder. Thus, we are obliged to replace the Cartesian, linear coordinates to which misinformed students are habituated by cylindric coordinates, with the understanding that each of the coordinates is part of a function of multiply-connected action. This carries us, in electromagnetism, for example, as far as Fourier Analysis.

Gauss carries us further. In place of cylindric coordinates, we have double-conical coordinates, expressing such included “elementary” functions as electrical potential, magnetic potential, and frequency, each and all multiply-connected. The multiple-connection of such conical (self-similar-spiral) coordinates is the generation of hyperbolic functions, as is the case in definition of *technology* in the science of physical economy.

The formal mathematical discontinuities generated by such hyperbolic functions might be termed “true singularities,” to distinguish their existence and physical significance from the simple singularities (point, line, solids, hyper-solids) of multiply-connected circular action. This organization of physical space-time is the minimal condition for representing adequately the universe in which we exist.

These singularities are everywhere dense, to such effect that their density is harmonically ordered in the manner Kepler ascribes harmonic functions to a physical space-time whose self-bounding curvature is con-

gruent with the harmonic orderings associated with the Golden Section. Indeed, the Golden Section is nothing other than the metrical characteristic of all projections of self-similar-spiral action upon a plane or onto a solid.

The generation of these singularities, and how space must be organized topologically to the effect that the continuity of action in physical space-time persists despite such singularities, is the subject of the successive work of Dirichlet, Riemann, and Weierstrass. This defines the mathematics of generalized non-linear functions.

This leads to the strongest of the theorems of Georg Cantor's elaboration of transfinite functions. The density of singularities within any interval of arbitrarily chosen smallness, of continuing non-linear action, is implicitly enumerable. A more adequate expression of that theorem conforms to a general theorem for relativistic physics.

The density of discontinuities per interval of action, as this occurs in the triply-connected conical coordinate system introduced as illustration, is the proper meaning of a measure of *physical potential*. Thus, in these terms of reference, we can construct *potential functions*, so defined, represented as surfaces in that triply-connected phase-space. These surfaces are surfaces of equal potential.

Singularities appear in a Riemann Surface as points, or topological singularities, as Dino de Paoli references this. The existence of those points, as topological singularities, poses a pair of interrelated problems. The first is more immediate, from the standpoint of what we have just reviewed; the second takes us directly to the matter of negative curvature.

"Points do not exist." Hence, a singularity in a Riemann Surface represents something other than a point as such. In part, this is already clear from the Riemann Surface Function itself. These points were generated by the kinds of hyperbolic functions associated with multiply-connected self-similar-spiral action, and are not to be apprehended as self-evidently existing points in any sense.

Our problem is, that in that form, we have left them represented as if they were points. We must recognize them as related to what de Paoli references as *caustics*.

To make short of it, once we treat these properly, as regions of negative curvature of physical space-time, the continued generation of the Riemann Surface Function to a higher order of topological connectivity, must follow.

This renders the unique ordering of the combinations of protons and neutrons, in the Periodic Table's array of elements and isotopes, most interesting, and also renders the so-called "neutrino problem" of nu-

clear-fission reactions most interesting. The necessity of the harmonically-ordered Archimedian-solid geometries, which coincide with that ordering of nuclei in the Periodic Table, if otherwise crucially demonstrated experimentally, shows, as de Paoli points toward this, that the apparent strong nuclear forces we must consider fall into place with the indicated role of Beltrami negative curvature as the characteristic of singularities in a Riemann Surface Function.

We are implicitly faced with an analogous state of affairs in the resolution of canonically lawful singularities generated by a quadruply-connected compositional process of classical polyphony.

Beethoven may not have been a specialist in the mathematical physics of the Gauss-Riemann domain, but he has, in a meaningful sense, mastered such principles in effect.

Music & Physics

We indicated above, that the fundamental progress of physical science requires us to move upward and backward historically at the same time. Backward, to search out those features of the internal history of science which account for the development of the ideas we must discard, and those to replace what we discard. In the search of the internal history of science, we must emphasize the study of the processes of the human mind, where ideas are generated, as much as the bounds of astrophysics, microphysics, and biophysics.

On this account, every advance in physics and related knowledge, must impel us to reexamine the creative processes' role in the creative features of classical fine art; at the same time we occupy our attention with the most profound and simplest of the crucial facts of physics qua physics.

This is much more than a formal requirement. If we compartmentalize ourselves to such effect, that one function of our mind, acting in one connection, is not efficiently aware of what the same function does in a different connection, we are to that degree schizophrenic, and everything we do partakes of a corresponding degree of schizophrenia.

The fallacy I attack is a widespread one, especially since the influence of Kant and Savigny fostered the idea of an hermetic separation of the methods and ontology of physical science from those of art and social practice generally. To save this civilization, we must end this false, schizophrenic dichotomy, and put the whole human being back together again, as if nothing different should ever have occurred.