Riemann's Crucial Insight

From Bernhard Riemann's habilitation dissertation, On the Hypotheses Which Lie at the Foundations of Geometry, translated by Henry S. White, in David Eugene Smith, ed., A Source Book in Mathematics (New York: Dover Publications, 1959):

It is well known that geometry presupposes not only the concept of space but also the first fundamental notions for constructions in space as given in advance. It gives only nominal definitions for them, while the essential means of determining them appear in the form of axioms. The relation of these presuppositions is left in the dark; one sees neither whether and in how far their connection is necessary, nor *a priori* whether it is possible.

From Euclid to Legendre, to name the most renowned of modern writers on geometry, this darkness has been lifted neither by the mathematicians nor by the philosophers who have labored upon it. The reason of this lay perhaps in the fact that the general concept of multiply extended magnitudes, in which spatial magnitudes are comprehended, has not been elaborated at all. Accordingly I have proposed to myself at first the problem of constructing the concept of a multiply extended magnitude out of general notions of quantity. . . .

[In conclusion:] This path leads out into the domain of another science, into the realm of physics, into which the nature of this present occasion forbids us to penetrate.