Chapter 2

J.S. Bach and inversion as a universal principle of development in the continuum of musical composition

by Jonathan Tennenbaum

In his article "The Substance of Morality," Lyndon LaRouche presents a conception of the Universe as a multiply-connected manifold of the type (*N*-manifold)/(*M*-manifold). "*M*" signifies an ever-expanding array of principles of development of human culture, and "*N*" signifies a growing array of principles of physical action. These two sub-manifolds, of order "*N*" and "*M,*" do not exist apart from each other, but are multiply-connected by Man's culturally-determined action upon the physical Universe, and the impact upon cultural development of changing physical conditions of human society's existence in the Universe. The inner developmental characteristic (curvature) of physical "*N*-manifold" is called *anti-entropy,* and the characteristic curvature of the "*M*-manifold" of human cultural development is *agapē.* The two are inseparable, necessary expressions of the Principle of Creation (God).

Before turning to the musical side of this question, it will be useful to clarify the meaning of "multiple-connectedness," and in what manner we are to conceive of a manifold that is governed by not one, but a growing multiplicity of developmental principles. To make a short work of this, I emphasize only some key points, followed by an elementary illustration from astronomy, which leads us directly to music.

It is impossible to reduce the relationship of events in multiply-connected manifold, by any deductive or similar means, to a single formal principle. Rather, action in the manifold is governed by a multiplicity of principles, none of which can be reduced to or derived from the others in a formal-deductive manner. Any process in the manifold is simultaneously co-shaped by each and all the principles in any arbitrarily small region of action. The active principles, mutually irreducible and incommensurable in the just-mentioned sense, constitute true *singularities*—individual existences underlying the whole structure of the manifold. We encounter such singularities in physics in the form of creative fundamental discoveries of principle, and in music as entirely analogous discoveries of principle of *bel canto*-anchored motivic thorough-composition. The following sections will review some of them, such as Haydn's discovery of *Motivführung,* and Mozart's breakthrough on the significance of the "Lydian" major/minor mode, first explored in the late works of J.S. Bach.

In first approximation, one might be tempted to think of each active principle as analogous to a coordinate axis in an *n*-dimensional space, *n* representing the number of an irreducible array of principles governing the manifold at a given stage of development. In reality, however, the principles of development, while mutually irreducible in a formal sense, are never independent of each other in the manner implied by the Cartesian coordinates or the use of "independent variables" in a formal mathematical representation. As an "*n*-manifold" develops to an "*n+1*-manifold" and so forth, the integration of each newly discovered principle *modifies* the entire previous array of active principles.

Indeed, this process invariably involves the generation of *paradoxes* and *anomalies:* events are demonstrated to occur in the Universe, which are incompatible with the given set "*n,*" point to a flaw or at least an inadequacy in that existing set of principles. Generally speaking, the newly hypothesized principle does not replace or supersede the existing ones; rather, the latter must be reworked and redefined from the standpoint of the new discovery. Thus, the process of lawful generation and resolution of dissonances through motivic cross-voice development in well-tempered polyphony, mirrors the universal features of development of any multiply-connected manifold.

The growing array of principles is subsumed within a higher principle of generation (a "One"), whose essential characteristic, anti-entropy/*agapē,* is located in the process of *change* from the lower- to the higher-order manifold. Although that process involves the successive integration of singularities, each formally incommensurable with the others, that higher principle of creative self-elaboration remains everywhere self-similar to itself. The proper measure of the ordering of development is not "number of dimensions" in the formal sense, but rather increasing *cardinality* or *power* in the sense developed by Georg Cantor.

The *ordering* of the process of development of the manifolds by increasing Cantorian cardinality, does not at all coincide with *time* in the ordinary chronological (i.e., clock-time) sense. On the contrary, *time* and *space* are merely subsumed physical principles, which are ironically multiply-connected with the Cantorian axis of development. We know this negatively, from the sad witness of rise and decline of civilizations or even human culture as a whole. We also know this positively, by the fact that all acts of creative discovery involve some or another degree of apparent "time reversal." Rigorous composition always proceeds *backwards* from the effect to be achieved—which exists, as it were, outside ordinary time—to the means and temporal pathway of events required to achieve that effect. Thus, music and drama are typified by ironic anticipations and premonitions, and other expressions of *temporal inversion.*

Music and Keplerian astronomy

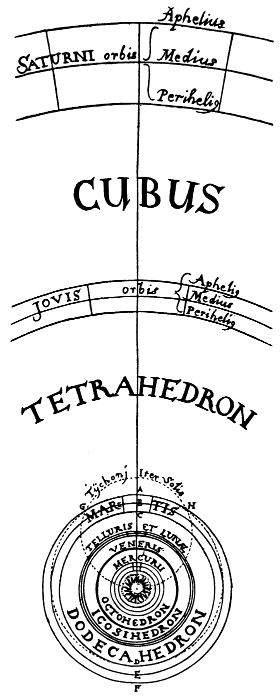
The development of astronomy, from the most ancient times up to Kepler and Gauss, provides the most direct access to the notion of a multiply-connected manifold, just sketched above.

The study of the motions in the heavens leads to the discovery of more and more *astronomical cycles* as *principles of motion.* Thus, observing the rising and setting of the Sun and the stars, we conceive the cycle of the *day.* Noting, however, that the path of the Sun shifts slightly from day to day, we discover the longer cycle of the *year.* What at first glance appear to be very slight discrepancies in the yearly cycle of the Sun with respect to the stars, reveal a much longer cycle of the *precession of the equinoxes.* Later, additional cycles emerge, connected with the non-uniform (elliptical) motion of the Earth around the Sun. In addition to these solar-terrestrial and stellar cycles, we must also take into account the cycles associated with the motions of the planets. The latter reveal themselves, upon closer examination, to involve more complex considerations, going beyond the principle of simple circular action.

Thus, as astronomy develops, we discover new principles of motion not only as new cycles per se, but also as internal principles of organization of the cycles, and principles of multiple-connectedness or "colligation" among the cycles. Thus, Kepler's discovery of the "area law" of motion in conic-section orbits, and his discovery of the harmonic principles underlying the entire array of orbits.

The observed motion of any planet or other heavenly body is the resultant of all cycles and related principles acting conjointly. So, for example, even though the equinoctial cycle has a length of some 26,000 years, it acts efficiently within any arbitrarily small time interval, to produce a *distinct,* implicitly measurable modification of any observed motion. The manner in which the characteristics of any planetary orbit are reflected in any arbitrarily small interval of the observed motion, was demonstrated by Carl Friedrich Gauss in 1801, when he determined the orbit of the unknown planet Ceres from only three, very close-spaced sightings.

FIGURE 2.1
Kepler's determination of the harmonic ordering of the solar system, from his *New Astronomy*



This concept of "curvature in the infinitely small" of astronomical motions, has an unavoidable, paradoxical feature: The motions we observe, embody not only the cycles which are *known* to us at any given time, but also those *we do not yet know* explicitly—cycles whose future discovery is inherent in the self-similarity of the principle of creation underlying the Universe as a whole. Hence, the curvature in the small, as reflected in the fine "articulation" of the heavenly motions, contains an element of *creative tension,* associated with the anti-entropy/*agapē* of a Universe constantly developing $M \rightarrow M+1, M+2, \ldots ; N \rightarrow N+1, N+2, \ldots$

As Kepler demonstrated in detail for the case of the solar system, the higher coherence of the astronomical "*n*-manifold," is reflected in *harmonic orderings,* of the same type as characterize *artistic beauty* in the domain of human Classical culture. In the dialogue *Timaeus,* Plato refers to this common higher principle underlying astronomy and Classical art, by declaring the Universe to be a continuously unfolding composition of "God the Composer."

Reflecting this, Kepler's determination of the harmonic ordering of the planetary orbits specifies certain band-like regions or corridors as the location of the planetary orbits, and not fixed algebraic values **(Figure 2.1).** The exact orbits of the planets, while remaining within their harmonically "quantized" corridors, are constantly changing and evolving together with the Universe as a whole, in a manner Kepler likened to the performance of a polyphonic composition.

Kepler's astronomical inversions

In his *New Astronomy,* Johannes Kepler presented a series of devastating anomalies which overturned the prevailing assumption, that the planetary motions were based on nothing but the Ptolemaic-Aristotelean notion of uniform circular motion as the basic physical principle. In order to determine the actual motions of the planets, however, Kepler had to overcome the difficulty, that the orbital motions of the planets, including of the Earth itself, cannot be adduced in any direct manner from the observed motions as they appear to an observer on the Earth. Indeed, as already remarked above, the *apparent* motion of any planet, is the resultant of a complex combination of motions, including the Earth's rotation, the Earth's motion around the Sun, and the true orbital motion of the planet. The true motion of the Earth around the Sun, which we can neither see nor sense in any direct way, can only be determined by reference to the actual motions of the other planets; but, to disentangle the real from the apparent motions of those planets, it would seem necessary to first know the motions of the Earth, from which we observe the planets. How do we get out of this circular paradox? Kepler's ingenious solution was based on a method of inversion, closely akin to J.S. Bach's method of well-tempered polyphony.

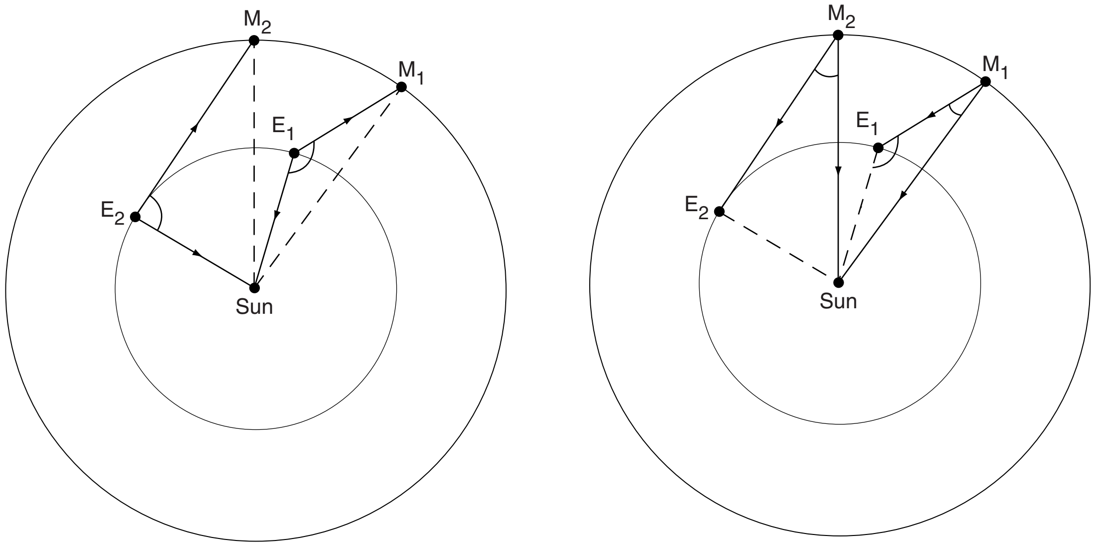
Kepler asked the hypothetical question: How would the Earth's motion appear, relative to the apparent motion of the Sun, if we were to observe the Earth and the Sun from Mars? An observer would have a different solar calendar, whose basic cycle (the Mars year) makes a specific ratio to the Earth year. At first glance, such a hypothetical shift of *locus of action*—analogous to a modulation or more general *inversion* in music, as we shall see below—seems only to compound our ignorance. Kepler, after all, had no means to actually place himself on Mars! Yet it was exactly by juxtaposing the motion of Mars as seen from the Earth, with the motion

FIGURE 2.2

Transformation of angular intervals by change of locus of observer

(a) Mars and the Sun, as observed from Earth

(b) Earth and the Sun, as observed from Mars



of the Earth as seen from Mars (all relative to the Sun as "tonic"), that Kepler was able for the first time to determine the orbits of *both* the Earth and Mars! **(Figure 2.2)** By thus exploiting the additional dimensionality provided by the Mars orbital cycle, Kepler was led to the discovery of the elliptical form of planetary orbits, and a revolution in astronomy. The key here is the *transformation* between two or more sets of angular intervals (e.g., observations referenced to the Earth's cycle, versus observations referenced to the Mars cycle).

Kepler was fully aware of the kinship of his method with the Platonic dialogue and the polyphonic principle in music. Just as one can only know one's own mind and the assumptions which shape it, in the mirror of our interaction with other minds; so, in well-tempered polyphony, the motivic idea emerges only through a process of contrapuntal inversions; and so in astronomy, the motion of our Earth would never be known—Kepler loved to say—had God not given us Mars and the other planets as celestial companions.

The well-tempered system, briefly

Turning to musical composition, remember that the "*n*-manifold" of musical development lies entirely *outside* the audible domain of musical tones per se. One might say, that musical ideas themselves *are soundless.* Yet these soundless entities generate all the events in the audible domain and rule over it absolutely. For example, as the performances of Wilhelm Furtwängler and Pablo Casals demonstrate most forcefully, a musical interval is not something determined by a pair of tones, like a line segment drawn to join two points. Rather, the interval *precedes* the tones, both ontologically and in the consciousness of the composer and great performer, just as the idea of the composition precedes the ordering and shaping of all intervals in a composition. Lyndon LaRouche emphasizes that the least "unit" coherent with the expression of a musical idea, is a *pair of intervals* in the sense of an *interval between intervals.*

The mere acoustician will puzzle over the paradox: What could be the difference between merely *playing tones,* playing intervals, and *playing the intervals between intervals,* in the manner Furtwängler brought his orchestras to do? Where resides the difference, given that the instruments themselves produce nothing but tones? The "extra" which distinguishes the performance of *intervals between intervals* from the mere sounding of a succession of tones, is clearly heard in the mind, but is otherwise a virtual *infinitesimal* in acoustical terms—often nothing more than a barely perceptible, specific shaping of the tones in a musical line.

That shaping of the tones by musical intervals, and intervals by intervals of intervals, embodies the same principle by which the well-tempered system as a whole is determined by the *curvature* of the evolving manifold of *bel canto*-based motivic thorough-composition. That development is bounded by the requirement, that the creative principle embodied in the conception of the *bel canto* singing voice, be extended in a self-similar manner to an *ensemble of bel canto voices* having differing registration. In this process, the harmonic principles of *bel canto* vocalization, investigated by Leonardo da Vinci and described in part in the preceding section of this report, are "turned inside-out," as it were, to become principles of well-tempered vocal polyphony.[1]

The result, evolving in the course of a long, implicitly still-ongoing historical development, is the Classical well-tempered system, with its various species of harmonic intervals (octaves, fifths, fourths, thirds, etc.).

The mature chorus and orchestra ensemble, as understood by Beethoven and Brahms, must sing as a *single voice,* even while performing the most intricately articulated polyphony. Conversely, instrumental and choral polyphony are nothing but a self-similar extension of the polyphonic principle inherent in the single *bel canto* human voice with its characteristic registral differentiation.

This is exactly the conception underlying Johannes Kepler's famous derivation of the musical intervals and scales, by harmonic division of the circle and sphere, which were crucial to his investigation of the musical principles governing the multiple-connectedness of the planetary orbits (see Kepler's *Harmony of the World,* Book III).

Unfortunately, Kepler's constructions are often misread to signify nearly the opposite of what they were originally intended to demonstrate.[2] The modern reader must never forget, that the *circle* and *sphere* of Kepler signify something very different from the mere geometrical shapes which carry the same names. Kepler explicitly refers to Nicolaus of Cusa, and the latter's discovery of the *ontological* significance of the circle's relationship, as a higher species, to its inscribed and circumscribed polygons. Cusa and Kepler stressed two elementary points in this context: First, the polygons and the discrete whole numbers associated with them, do not exist self-evidently, apart from circular action; and there is no valid determination of the polygons which does not originate in the circle. Second, while the polygons are generated and everywhere bounded by circular action, it is impossible to go backwards and derive the circle from the polygons, even if the number of their sides were increased beyond any limit.

Exactly in this sense, the generative principle or *curvature* of the *n*-manifold of *bel canto*-based motivic thorough-composition, bounds the process of successive discovery of principles of composition, including the system of harmonic intervals, tuning, keys, modes, and everything else. There can be no self-evident algebraic determination of musical intervals, nor any valid construction based on "empirical facts" concerning acoustics and the physiology of hearing, as Helmholtz claimed. The well-tempered system is everywhere bounded by the creative process of musical development.

Thus, contrary to a nearly universal misunderstanding, the well-tempered system not only does not prescribe an algebraically-fixed set of pitches and intervals, but it *absolutely forbids any such "fixing"! Bel canto*-based well-tempered composition dictates the *necessity* for a specific "shaping" of each and every tone and interval in a composition—including lawful *variations of pitch* within the harmonically-ordered "corridors" identified with the scale-steps, in such a way that the infinitesimal "curvature" of each moment of articulation expresses the creative tension underlying the composition as a whole.[3] Unfortunately, the capability of distinguishing such small but crucial nuances, possessed by composers and to a large extent even the educated musical audiences of Beethoven's time, has virtually died out.

By contrast, the concept of strict mathematical equal-tempering, is a fallacy rooted in the vain attempts to collapse a multiply-connected manifold into the "flat" space of a single (monophonic) formal principle.

Inversion of intervals

As indicated, *inversion* is a universal principle of musical development. To gain some insight into this, we can start by examining the manner in which Classical composers employ elementary inversions of *intervals* as instrumentalities of the process of motivic-polyphonic develop-

FIGURE 2.3a

J.S. Bach, *Jesu, meine Freude,* opening chorale

FIGURE 2.3b

Schematic of J.S. Bach, *Jesu, meine Freude,* opening chorale

ment. As we move forward in this series of articles, we will work upward from the simplest cases, discussed here, to the higher conception of inversion which underlies the late compositions of Mozart and Beethoven. In the process, we must constantly reflect on the way our minds "hear" both the explicitly stated intervals, and those which are only implied by the composer, and which are often even more important than the stated ones. These distinctions, reflecting changes in assumption governing any given phase of a composition, must be expressed in performance, by the articulation and "shaping" of tones and intervals "in the small" (including lawful nuances in pitch intonation).

In its very simplest formal manifestations, inversion involves one of three forms of *transformation* of an interval subsuming two tones:

(1) By sounding one or both of the tones in a different octave, voice, or register; usually in such a way, that the higher of the two becomes the lower in pitch, and the lower becomes the higher, while retaining their values within the scale (inversion of order in pitch). So, for example, a soprano and bass singer. In this case, the magnitude of the interval is changed; a fifth becomes a fourth, a major third becomes a minor sixth, and so forth.

(2) By reversing the *direction* of the interval's motion, as taken from either of the tones regarded as the origin, i.e., from upward to downward and vice-versa, while retaining the relative magnitude of the interval. In this case, not only the relation of higher and lower in pitch is reversed, but also the scale-value of one of the tones. So, the fifth from middle c′ *upward* to the g′ above it, inverts to the *downward* fifth from middle c′ to the f below it. (Note: this kind of inversion is more than a simple transposition of the interval; the *directionality* is also changed.)

(3) By reversing the *temporal order* of the two consecutive tones, so that the later now becomes the earlier, and vice versa, while maintaining their pitch values.

It is important to bear several things in mind: First, each event of inversion, constituting a *transformation of intervals,* involves no less than a *pair* of intervals—the original and its inversion. Inversion can thus be considered as a special type of *interval between intervals.* In many cases (see below) the original interval is merely implied, but not explicitly stated; or vice-versa, the original interval may be stated explicitly, and the inversion only implied. Sometimes *neither* of the two are stated explicitly, but are unmistakably implied. Related to this, inversions can occur for intervals which span entire sections of a composition, rather than merely consecutive tones, and so forth.

Now let us look at some examples of elementary forms of inversion in compositions of J.S. Bach. I want to emphasize that the following remarks by no means amount to an adequate analysis of any of these compositions. They are intended to open doors for an appreciation of the role of inversion in composition, starting from the very simplest sorts of cases, and working upward toward the more complex and profound.

J.S. Bach's motet *Jesu, meine Freude*

The first two measures of the opening chorale of J.S. Bach's motet *Jesu, meine Freude* (**Figures 2.3a and 2.3b**), present us with an anomaly: The soprano voice describes in stepwise motion, the *descending fifth* b′-e′, while the bass moves downward from e, and then back to e. Consistent with this and the motion of the inner voices, we hear e as the base-tone and the downward fifth b′-e′ as a return (from where?) to the base tone. The whole motion of the voices is more like the *end* of a statement, than the beginning.

Now glance at the intervening development. From measure 3 to the beginning of measure 4, the soprano voice goes upward from b′ to e″, spanning an *upward fourth* b′-e″, which is the *inversion* of the *downward fifth* b′-e′ of the initial measures. Thereby, in our mind we "hear" the octave e′-e″ as confirming an implicit development e′-b′-e″, in which the first interval has been time-inverted in the opening statement.

The movement e′-b′-e″ would have achieved a certain closure, but that the soprano, instead of resting at the newly gained e″, falls back to the adjacent d♯″; while the bass voice articulates the *upward fifth* e-b, which is an *inversion* of the soprano's *descending fifth* in measure 1. At this point, we reach a maximum tension, associated with the unresolved juxtaposition of the intervals b′-d♯″, b-f♯′ (in the tenor voice), and the expected closure e′-e″. The resolution to e′-e″ is achieved in measures 5-6, by the soprano—anticipated already by the tenor's motion in measure 4—breaking into the third register to reach e″ from above, via the g″ and f♯″.

With the consolidation of the octave e′-e″, the chorale moves downward to its conclusion. Then, after some preparation in measures 13-15 (themselves expressing an inversion), in measures 16-17 the soprano moves stepwise down to b′ (inverting the upward fourth b′-e″ of measures 3-4), from which it descends the remaining downward fifth b′-e′ to close the descending octave e″-e′ and end the chorale. That downward fifth, quoting the initial statement of the chorale—albeit with a shift in meter and a significant change in the tenor voice—resolves the original paradox: The beginning originated from the end!

All of this is nothing more than the most elementary kind of intervalic inversion, associated with the natural strophic organization of the chorale. The point is to see how the counterpoint developed by Bach in the bass and inner voices, defines and brings out the *changes in meaning* associated with the indicated inversions of what is at first glance one and the same interval.

J.S. Bach's *The Art of the Fugue*

We concentrate first on just a few measures of the opening statement of the fugue. (See **Figure 2.4a** for the entire fugue, and **Figure 2.4b** for conceptual sketches of it.) What we are about to point out would be immediately perceived by any musical audience in Beethoven's time. Today, however, the same things would pass unnoticed by most listeners, on account of their lack of grounding in composition. Hence the need for the following, relatively minute examination.

The fugue begins with a first statement of the theme (measures 1-5) with an initial contrapuntal elaboration through measure 8. Our initial hearing of the fugal theme is dominated by the statement of the *upward fifth* d′-a′ in measure 1. In measure 2, the upward motion is reversed; a *downward third* f′-d′ is stated, closing back to what we have already sensed to be the base-tone (tonic) D. At that moment, we "hear" in our mind *two additional, implied intervals:* first, an implied *unison* between the initial d′ of the first measure and the final d′ of the second measure; and second, an implied *downward fifth* a′-d′, which is the reversal of the upward fifth d′-a′ of the first measure. This is the *first reversal/inversion.*

Our initial hearing of the following two measures 3 and 4, is dominated by the *down-*

FIGURE 2.4a
J.S. Bach, Fugue I from *The Art of the Fugue*

continued on following page

FIGURE 2.4a *(continued)*

ward half-step d′-c♯′, from the end of measure 2 to the beginning of measure 3, and the fact that the reversal of that interval (i.e., c♯′-d′), which the earlier reversal d′-a′, a′-d′ makes us expect to hear, is not really accomplished until we reach d′ in measure 5. Indeed, although c♯′-d′ occurs nominally already in measure 3, the d′ is sounded off the beat, as a quarter note—too short and with too much the character of a passing note, to fully resolve the preceding d′-c♯′, which was stated strongly in half-notes and with c♯′ on the beat. In any case, the relationship between d′-c♯′ (measure 2 and 3), and the c♯′-d′ implied between the c♯′ of measure 3 and the d′ at the beginning of measure 5, constitutes a *second inversion.*

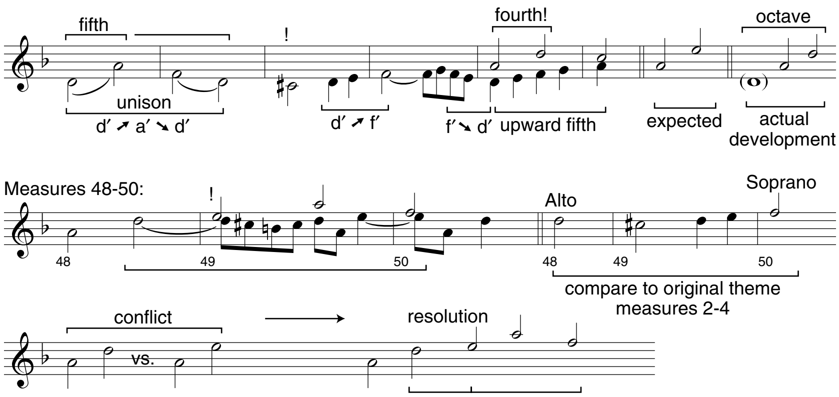
The intervening passage, from measure 3 to the beginning of measure 5, is somewhat inconclusive at first hearing; what stands out is a *third reversal/inversion* implied between the sequences c♯′-d′-e′-f′ *upward,* g′-f′-e′-d′ *downward,* in measures 3 and 4 respectively. The latter is clearly heard as quoting the *downward third* f′-d′ of measure 2, and the former as stating its reversal/inversion. However, the sense of reversal is "modulated" by the intervention of neighboring tones c♯′ in the first case and g′ in the second, plus the syncopation and acceleration of motion. Many things are suggested by this articulation, which are only actualized later in the fugue, and in later fugues of the entire *Art of the Fugue* cycle. Finally, note that all three reversals/inversions pivot on the common d′ (as, in a sense, a pedal-point), strengthening our sense of d′ as the hypothesized pivot or base-point of the whole composition.

All of this is preparatory to the second entrance of the theme, in measure 5. At the sounding of the a′, we immediately "hear" an implied unison with the a′ of measure 1, and recall the initial upward fifth d′-a′, which the initial (lower) voice now once again quotes in stepwise motion from the beginning of measure 5 to the beginning of measure 6.

At this point a potential conflict appears.

In the original theme, the upward fifth d′-a′ subsumes an implied register shift (relative to soprano registration), from first to second register, establishing a′ initially as the dominant tone in that register. This already creates the sense of a′ as a second potential pivot-point or focus of the developmental action. This potential focal-point function is strengthened by the reversed pair of intervals d′-a′ upward, a′-d′ downward, which can be heard from the standpoint of either d′ or a′ as the pivot-point. As

FIGURE 2.4b
Two key passages from Fugue I of J.S. Bach's *Art of the Fugue*



a result, our mental ear already "hears" as a strong implication the *upward fifth* a′-e″; it is implied as the inversion of the *downward fifth* a′-d′ and as the transposed quotation of the *upward fifth* of the fugal theme to a′ as a new focus.

On the other hand, other strong reasons point to an *upward fourth* a′-d″ as the lawful sequel at this point. In fact, if d′ remains the focal-point, then already the first upward fifth d′-a′ in the very first statement of the fugue, calls for its continuation in the upward fourth a′-d″, which would thereby complete the octave d′-d″. In this way, the original reversal, namely:

> d′-a′ (upward fifth) reversed to a′-d′ (downward fifth) in the statement,

would be quoted via the second voice as:

> d′-a′ (*upward fifth* implied by placing the second entrance the theme at a′, against d′ as a pedal-point), inverted to the *upward fourth* a′-d″ as the first interval stated by the second voice.

In other words, the original motion d′-a′-d′ becomes d′-a′-d″.

We therefore have a *dissonance* between two (as yet unheard!) inverted intervals: an upward fifth a′-e″, and an upward fourth a′-d″, both of which are inversions of the interval that commences the composition. Which of them will actually occur? Bach, in fact, chooses a′-d″; but the dissonance with the implicit a′-e″ is still heard in the mind, and acts to drive the development forward.

Let us briefly note some features of the rest of the fugue, which confirm this reading of the indicated passage.

First, the upward fourth a′-d″, which is an inversion of the original interval of the theme d′-a′, becomes the ever-more-dominant motif throughout the subsequent development. It is first echoed in the counterpoint in measure 7, and is taken up as a subsumed motif in subsequent counterpoints and especially the interludes of measures 17-21, 36-40, and 44-46. It evolves into the increasingly powerful counterpoints of the second half of the fugue, in the turning-point in measures 48-53 (and sequel), as well as the final development of measures 66-70, leading to the coda, where it is compressed to a new figure in the soprano voice.

Second, the turning-point beginning in measures 48 and 49. Here, the original upward fourth a′-d″ is stated again, as if to repeat the fugal statement at a′. But the listener is surprised: what follows instead is the dramatic entrance of the soprano voice with a second upward fourth e″-a″, reaching into the soprano's third register and initiating the statement of the fugal theme in the highest register-range of the fugue. The dissonance between the intervals a′-d″ and a′-e″, now explicitly re-created by the juxtaposition of the fourths a′-d″, e″-a″, is effectively resolved by the f″ in the upper voice of measure 50, by completing the upward sequence d″-e″-f″. Note how the alto counterpoint adds c♯″ to yield c♯″-d″-e″-f″, which is exactly the original statement of the third measure of the fugal theme, stated one octave higher.

J.S. Bach's *Mass in B minor*

One of Bach's most condensed masterpieces of vocal counterpoint, is the six-part double fugue "Gratias agimus tibi" in the "Gloria" of Bach's *Mass in B minor* (the four-part vocal chorus is expanded, in the second half of the fugue, by two trumpet voices). The same figure recurs in slightly altered form as the final section of the mass, "Dona nobis pacem."

The initial statements of the fugue **(Figure 2.5)**, introduced in a "canon of canons" between the two sets of voices (bass-tenor and alto-soprano), appear at first glance to be dominated by the notion of d as the base-tone, the upward major third d-f♯ and fourth d-g in the fugal theme, and the rising fifth d-a between the bass entry and tenor entry. The inversion a-d′ of the latter interval, is stated by the tenor (measure 2) itself, and between the tenor and alto entry in the same measure, and is then repeated in different registration between the alto and soprano entries (measures 2 and 3).

However, the rhythmic and contrapuntal arrangement of the voices also implies *other* intervals and inversions. Prominent among these are the intervals d′-b and b-d′, implied, for example, in the tenor voice line (measure 2) and variously in other registers between the bass, tenor, and soprano (measures 3 and 4). All these intervals are heard in the initial section essentially from the standpoint of d as a kind of pedal-point. However, beginning in measure 10, and decisively in measure 13, the appearance of b in the bass line, redefines the entire set of relationships, now obliging us to hear the original sequence d-e-f♯-g from a completely different standpoint, defined by an *inversion* of the original relationship of d and b which places b as the pivot in the bass (see also the discussion below).

The moment of this redefinition coincides with an implied bringing-together of the two double-fugal themes, already implied by the bass line's reference to the second fugal theme in measures 10 and 11, and in measure 13. The basic movement of the second theme, which is first stated in measures 5-6 and recurs in various major/minor variations in the course of the fugue, is the repeated initial note, followed by a cascade of sixteenth notes which elaborate a descending sequence a-g-f♯-e. This is referenced by the bass with its descending sequence b-a-g♯-f♯ in measures 10 and 11, sounded against the rising major tetrachord d′-e′-f♯′-g′ in the alto. The second reference is in measures 13-15, where the descending scale steps are inverted to b-d in the elaboration by eighth notes. The second reference confirms the first one.

The special significance of this juxtaposition of the ascending major tetrachord d-e-f♯-g against the descending minor b-a-g-f♯, is that the two sequences are *exact inversions of each other:* they contain the same sequence of steps, but in the opposite directions.

The same inversion is reaffirmed at a crucial transition, in measures 25-27 (not shown), where the bass voice's descending motion is continued to a decisive statement of b-d, thereby reversing the original inversion d-b and restoring d as the basetone.

This fugue is a good example of the absurdity of all formal definitions of "key." To the question, in what key the fugue is actually written, the textbook answer would be, "in D major, of course." Yet, such an answer is incompatible with the entire effect of the fugue. Nor could we characterize the composition adequately by simply calling it B minor. It were more accurate to speak of a B minor seen through D, or a D major/B minor "mode," developing through inversions around the interval b-d.

Inversion and the Lydian/major-minor mode

To conclude this discussion of inversion, let us look ahead toward the genesis of the more advanced conception which is exemplified by such later works as Mozart's C major/minor Fantasy K. 475, and Beethoven's late string quartets.

Those compositions embody a fundamental discovery, which integrates the major and minor modes of the well-tempered system into a *new principle of composition,* sometimes called the "Lydian major/minor mode." That discovery, which involves not one, but many principles of composition, will be elucidated from various angles in the following sections. Here, we focus on one aspect of the relationship of major/minor with the principle of inversion.

First, we should emphasize, that the entities we call "keys" and "modes" are not formal constructs, but—to the extent they mean anything at all—signify sets of assumptions or hypotheses governing specific phases of composition. The difficulty is, that the assumptions involved, cannot be identified with specific scales or other literal feature in some formal, "algebraic" fashion. Thus, it is easy to demonstrate that the most elementary and ubiquitous features of J.S. Bach's music are incomprehensible from the standpoint of any formal notion of musical key. The assumptions and hypotheses are not located in the notes, but in the thinking process "behind the notes."

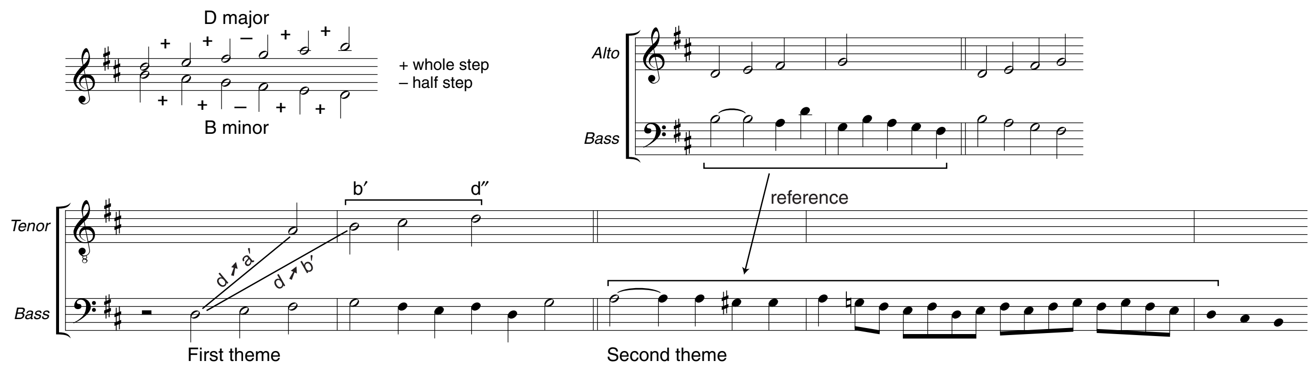
That said, the characteristic distinction of hypothesis between (for example) C major and C minor would seem to lie in the

FIGURE 2.5a

J.S. Bach, 'Gratias' from *Mass in B minor*

FIGURE 2.5b

Schematic of ‘Gratias’ from J.S. Bach, *Mass in B minor*



different manner of forming *thirds* from C and its closest relations, F and G. So, C major features the major thirds (more appropriately termed in German “*große Terzen*” or “great thirds”) C-E, F-A, G-B; while C minor features the “*kleine Terzen*” (“small thirds”) C-E♭, F-A♭, G-B♭. Looking at the intervals between these intervals, note that E, A, and B are neighbors by fifths, as are E♭, A♭, and B♭. Moreover, the first group is related to C by a series of successive upward fifths:

C-G-d-a-e′-b′,

while the second group is related to C by a downward or inverted series of fifths (i.e., by fourths):

c″-f′-b♭-e♭-A♭.

Related to this, the *downward* tetrachord in C minor, C-B♭-A♭-G, is the exact inversion of the *upward* tetrachord in C major: C-D-E-F. In this sense, C major and C minor appear related to each other by a series of inversions.

The crucial missing singularity, needed to bring together C major and C minor in a closer unity, is expressed in F, or rather the *interval* C-F, which is the pivotal singularity of the whole musical system, corresponding to the arithmetic-geometric mean of the octave C-c and the anchor for the whole array of *bel canto* register-shifts. If we adjoin F, then we obtain a notion of C major/minor as a multiply-connected manifold which “grows” from c′ in both directions, ascending and descending fifths, as follows:

(ascending)
C → G → d → a → e′ → b′ → f♯″

(descending)
c‴→ f″→ b♭′→ e♭′→ a♭→ d → G♭,

the latter (F♯ and G♭) belonging to the same tonal corridor in the well-tempered system. Thus, the coherence and connectivity of the manifold lies in the so-called Lydian interval, C-F♯, which is the anchor of our new major-minor mode.

The upper set of tones forms a scale

C-D-E-F♯-G-A-B-c,

which coincides with the so-called *Lydian mode* in the ancient Greek musical system, and is characterized by the crucial interval C-F♯. The lower set of tones forms a second scale

C-D♭-E♭-F-G♭-A♭-B♭-c

which is the exact inversion of the first, Lydian scale.

Now, some might shrug their shoulders at this, pointing out that all this is nothing more than the “circle of fifths” producing a perfectly symmetrical, chromatic, twelve-tone scale. This absurd conclusion completely ignores the *bel canto* principles determining a non-algebraic, non-equal-tempered system, principles which determine the unique, pivotal role of C-F♯.

The result, as the use by Mozart and Beethoven of this “Lydian major/minor mode” demonstrates, is not to *lessen* the sense of *tonality* in music, but actually to greatly *strengthen* it. This remark is extremely important, owing to the widespread, but totally fallacious claim, that Classical music evolved “naturally” toward the atonal cacophony of so-called modern music. In fact, far from being a step *toward* arbitrary chromaticism, the C-F♯-based Lydian mode, as understood by Mozart and Beethoven, achieves an enormous increase in the “Cantorian” ordering-power of tonal composition. Thereby it became possible to eliminate any remnants of arbitrary chromaticism that might otherwise be hiding between the toes of the earlier major-minor system.

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1. What is commonly referred to as “melody,” including so-called solo melody, is nothing but a derived feature of vocal polyphony. Strictly speaking, *monophonic melody does not exist.* What we call the melody of a solo voice, for example, is nothing but that voice’s singing of an intrinsically polyphonic composition. A relevant reflection of J.S. Bach’s views on the polyphonic principles of so-called melodic (or better, motivic) development, is contained in the first biography of Bach, written by Nicolaus Forkel [“On Johann Sebastian Bach’s Life, Genius, and Works,” in The Bach Reader, ed. by Hans T. David and Arthur Mendel (New York: W.W. Norton, 1966)]. Otherwise, the cases of Gustav Mahler and Richard Wagner typify the way in which, as soon as composers depart from the rigorous principles of well-tempered polyphony, their melodies degenerate into nothing but ugly groaning.

2. In Book III of his *Harmony of the World,* Kepler polemicized against the empiricist, mechanical theory of musical consonance and dissonance, which had been put forward by Vincenzo Galileo, the father of Galileo Galilei. Vincenzo is regarded as the pioneer of the reductionist musical theory later associated with Jean Le Rond d’Alembert (1718-1783) and Jean-Philippe Rameau (1683-1764), which became virtually hegemonic by the end of the Nineteenth Century, thanks to Hermann Helmholtz (1821-1894).

3. Further exploration of this point might usefully focus on the significance of vibrato in the *bel canto* singing voice—a vibrato which, in strong contrast to the Romantic’s pathetic tremolo, is defined as a variation of pitch within a well-tempered pitch-corridor. Apart from the role of vibrato in the technique of *bel canto* singing, one can demonstrate how passages sung without the vibrato, i.e., at a “mathematically fixed” pitch, are correctly heard as *wrong,* destroying the fabric of explicit and implied cross-voice relationships.