

EIR Feature

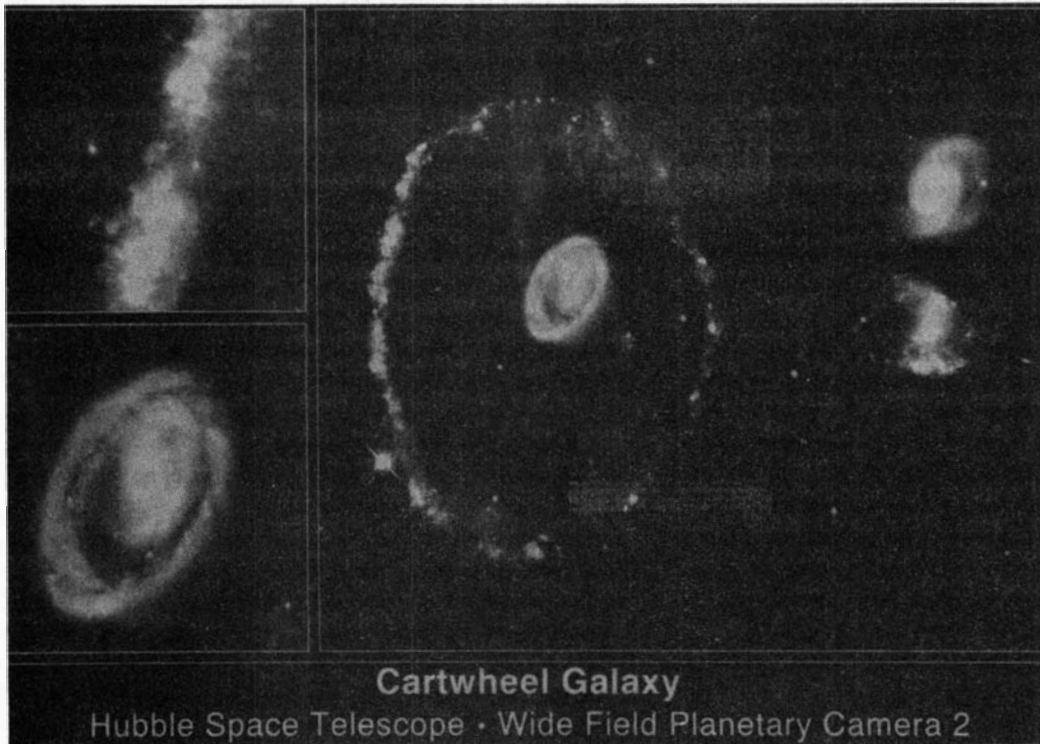
Georg Cantor's contribution to a new renaissance

by Dino de Paoli

This speech was delivered at a conference of the Schiller Institute in Halle, Germany on May 6, 1995. George Gregory translated the German quotes from Cantor, Leibniz, and others.

This year marks the 150th anniversary of Georg Cantor's birth, and the 100th anniversary of his discovery of the "paradox of the Absolute." Cantor is one of the greatest German mathematicians of the nineteenth century, and his discoveries are in a difficult area of mathematics, whose formalism would be irrelevant to this audience. On the other side, modern "Cantorian mathematicians" tend not only to forget the background of Cantor's discoveries, but also to *oppose* the philosophy which led to such breakthroughs. For this reason, we will tell the story of "a mathematician who did not want to be one," the story of a man whose real identity and inspiration were more on the artistic and philosophical side of his personality, the side that he loved the most; we will also show the relation between his outlook and his insights in criticizing a certain cultural trend in Germany.

At the end of the nineteenth century, Europe became increasingly dominated by cultural decadence and pessimism, whose danger became obvious, unfortunately, only much later. Different trends were preparing people to accept totalitarianism as a "scientific" way of life. Naturalism was presented as a new "scientific" religion, whose actual aim was to politically and economically justify the reduction of man to a mere beast. In this sense, it was important to destroy Judeo-Christian religion, which stressed the sacredness and the "image of God" in man. This had to be presented as a reactionary mythology, disproved by the "science" of Darwin and Malthus, science which, in reality, was coherent with "the culture of the empires." This "Conservative Revolution," past and present, always covers evil with a "theory" of some type of Social Darwinism: the survival of the strongest "race," the hatred of whatever "other" is considered to be threatening our "living space." In that culture of the *fin de siècle*, the deep Christian humanism of a



A view from the Hubble Space Telescope of a head-on collision between two galaxies (right). Said Lyndon LaRouche, in his speech to the Schiller Institute's seminar: "Cantor emphasizes that the universe that exists, is the universe of the Becoming, not a fixed, empty space-time with objects floating around in it. And that is, he says, the Transfinite, by which he means, inclusively, what becomes known as the Aleph series. He identifies the Good of Plato as his notion of the Absolute, of God."

Beethoven or a Schiller seemed of another world. Few people, and no mathematicians, saw the evil coming, or reacted, when to degenerate, was named "to progress," and to be pagan, was said to be "modern."

Cantor, in his way, fought against such a "world of the beasts." For these reasons, I would start our discussion of Cantor's life with a Schiller poem which best sums up his free creative life and his work, "Die Worte des Glaubens" ("Words of Faith"—see page 27).

Cantor's life

G. Cantor was born on March 3, 1845 in St. Petersburg, to Georg Woldemar Cantor and Marie Boehm. Cantor wrote about his father: "My father . . . came to St. Petersburg as a child, and was soon christened into the Lutheran Church there. But he was born in Copenhagen of Israelite parents, who belonged to the Portuguese Jewish community there." (M 380)¹

For Cantor, clearly his father's family was of Jewish origin, but for some strange reason, there are "experts" today who try to deny this; we shall see, later on, the importance of the issue. In any case, according to Cantor's son, the Nazis requested that "the marble bust of Cantor be removed from the main building at the University of Halle."

His mother came from a very well-known Catholic family. Cantor wrote: "In fact, I belong to a family of violin virtuosos on my mother's side. My grandfather and my grandmother . . . entertained musical circles in Petersburg

as royal violin virtuosos; and my great-uncle Joseph Boehm leads a conservatory in Vienna and is the founder of a famous

In this section

At a seminar in the German city of Halle on May 6, the Schiller Institute paid tribute to the great German mathematician Georg Cantor, who made Halle his home. We publish here the speeches given there by Dino de Paoli, Lyndon LaRouche, and Jonathan Tenenbaum.

For more information on Cantor, see:

Georg Cantor, "Foundations of a General Theory of Manifolds," first English translation of "*Grundlagen einer Allgemeinen Mannigfaltigkeitslehre*," *The Campaigner*, January-February 1976.

Lyndon H. LaRouche, Jr., *The Science of Christian Economy* (Washington, D.C.: Schiller Institute, 1991).

Lyndon H. LaRouche, Jr., "Georg Cantor: The Next Century," *Fidelio*, Fall 1994.

Dino de Paoli, "Georg Cantor's Contribution to the Study of Human Mind," *21st Century Science & Technology*, Summer 1991.

school for violinists.” (M 4) Joseph was indeed very close to Beethoven; he played many of his works, such as, in 1825, the Quartet in E-flat Major, Op. 127. Marie herself was also a musical talent, as was Georg Cantor’s brother Constantine, who became a very good pianist. Cantor’s sister Sophie was instead very good at drawing. Georg seems to have had qualities in both artistic domains. As a musician, he described himself this way:

“In a profound sense, I have a very facile artistic nature, and I have always regretted, that my father did not allow me to become a ‘violinist,’ for at that, at least, I would have been happy.” (M 4) “I myself began to play the violin when I was only 6 years old, and made quite some progress; it was only the peculiar circumstance which came about when I reached the age of 16, I myself no longer know how, when I made mathematics my profession, that took me away from this happy profession as an artist, and for 30 years now, my violin has been lying forlorn and rotting in a dusty case, only there to awaken in me from time to time the fleeting doubt, whether I might not have been happier, had I remained loyal to it. . . .” (M 416)

As for his drawing, we can judge directly by observing his only extant work, the drawing of a dog (Cantor was 17). Nevertheless, as he said, he followed another career.

The family moved in 1856 to Germany, near Frankfurt am Main. He attended the Gymnasium in Wiesbaden, the Realschule in Darmstadt, in 1862 the University of Zurich,



Pencil drawing by Cantor, dated 1862. The artistic side of his personality was the side Cantor loved the most.

and in 1863 the University of Berlin. There, in 1867, he completed his doctoral dissertation, and in 1869 he had his Habilitation in Halle, where he became assistant professor and, in 1872, special professor. He stayed in Halle until his death on Jan. 6, 1918.

In 1874, he became engaged to Vally Guttmann, and married her the same year. “My dear little Vally,” as he called her, was also very musical, studied at a conservatory, and was ranked as one of the best students of piano and voice. With Vally, Cantor’s home became a familiar place for evening music recitals, celebrations, and philosophical discussions. Vally’s professional singing is recorded, for example, in the Program of the Singakademie for March 20, 1889:

1) Sonata (D Major) for two pianos, W. Mozart (Frau Prof. Cantor and Frau Prof. Stumpf)

2) Duet from “Figaro” (Frau Prof. Sucher and Frau Prof. Cantor)

3) Andante and Variations for two pianos, Schumann (Frau Prof. Cantor and Frau Prof. Stumpf).

The Cantors had six children: Else, Gertrud, Erich, Annemarie, Margarethe, and Rudolf (the youngest, born in 1887). Else became a famous “singer and acknowledged music teacher.” Rudolf, his youngest and most beloved son, died at age 12 in 1899. The event sent Cantor into a deep crisis. He wrote:

“The child was indeed tender and frail in his first year . . . but for six years he has developed physically and mentally to the best, and was so loving and lovable, that he became the favorite of the whole family. He was extraordinarily talented musically, so that I succumbed to the hope that he would step into the tradition of the Boehm family on my mother’s side. . . . Thus did the wish emerge in me, that Rudolf should dedicate himself entirely to music, which had made him so happy even in his early youth. And now this hope is gone!”

In 1879, Cantor became a tenured professor in Halle, and until 1882-83 he achieved the most important mathematical discoveries, which culminated with the publication of his *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (*Foundations of a General Theory of Manifolds*).

Soon after the publication of this highly philosophical type of mathematics, in which he elaborated the invention of the “transfinite” numbers, Cantor was subjected to brutal hostility from “the Berliners,” led in particular by Leopold Kronecker. Precisely in the same year, Cantor had the first symptoms of a nervous breakdown, whose origins are unclear, although it was reinforced by disappointments, pressures, and hostility. That summer, the attack was very short, but later it became progressively more intense and recurrent, especially after 1900. Only during such periods of sickness was he involved in his strange research to prove that Francis Bacon was the real Shakespeare.

In 1885, Cantor started to support the fundamentals of his discoveries by intense study of philosophy and theology. In November 1884, he had started close contacts with J.B.

Die Worte des Glaubens

Friedrich Schiller

Drei Worte nenn ich euch, inhaltschwer,
Sie gehen von Munde zu Munde,
Doch stammen sie nicht von außen her,
Das Herz nur gibt davon Kunde.
Dem Menschen ist aller Wert geraubt,
Wenn er nicht mehr an die drei Worte glaubt.

Der Mensch ist frei geschaffen, ist frei,
Und würd er in Ketten geboren,
Laßt euch nicht irren des Pöbels Geschrei,
Nicht den Mißbrauch rasender Toren.
Vor dem Sklaven, wenn er die Kette bricht,
Vor dem freien Menschen erzittert nicht.

Und die Tugend, sie ist kein leerer Schall,
Der Mensch kann sie üben im Leben,
Und sollt er auch straucheln überall,
Er kann nach der göttlichen streben,
Und was kein Verstand der Verständigen sieht,
Das übet in Einfalt ein kindlich Gemüt.

Und ein Gott ist, ein heiliger Wille lebt,
Wie auch der menschliche wanke,
Hoch über der Zeit und dem Raume webt
Lebendig der höchste Gedanke,
Und ob alles in ewigem Wechsel kreist,
Es beharret im Wechsel ein ruhiger Geist.

Die drei Worte bewahret euch, inhaltschwer,
Sie pflanzt von Munde zu Munde,
Und stammen sie gleich nicht von außen her,
Euer Innres gibt davon Kunde,
Dem Menschen ist nimmer sein Wert geraubt,
Solang er noch an die drei Worte glaubt.

Words of Faith

I'll name you three content-laden words;
From mouth to mouth they are chasing,
But not from outside of us do they emerge—
'Tis words from the heart we are facing.
Mankind is of all his value bereft
If in these three words no faith is left.

Man was created free—*is* free
E'en though he were born in shackles.
Do not be deceived by the rabble's bray
Or idiots' abusive cackles.
Before the slave, from his chains uncaught,
Before man set free, O tremble not!

And virtue—this is no meaningless sound—
Can be practiced each day if we trouble;
And much as we tend to go stumbling around,
Toward paradise, too, can we struggle.
And what no logician's logic can see
The child-like mind sees obviously.

And one God there is, a Will divine,
However man's own will may waver;
Supremely above all space and all time
The living Idea moves forever.
And though all's e'er-changing in form and in scene,
Within that change rests a spirit serene.

Keep these three content-laden words;
From mouth to mouth implant them.
And if from without they do not emerge,
Then your innermost soul must grant them.
Mankind is never of value bereft
As long as his faith in these three words is left.

—Translated by John Sigerson

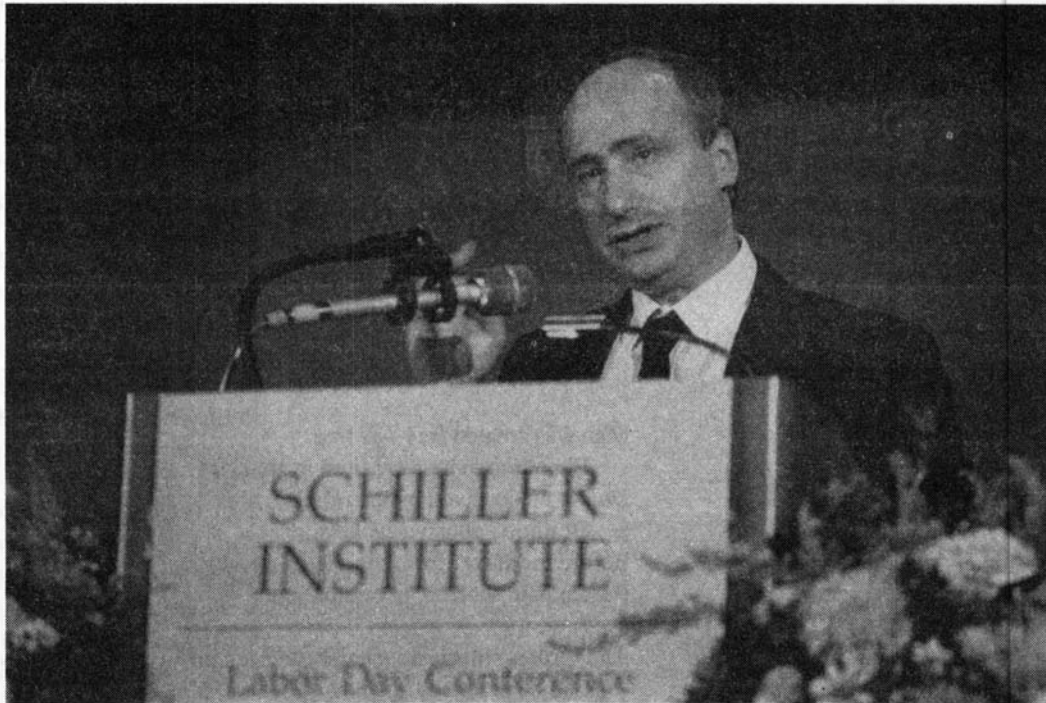
Published in the Schiller Institute's *Friedrich Schiller: Poet of Freedom*, Vol. I (New York: New Benjamin Franklin House, 1985).

Cardinal Franzelin in Rome, and with other, mainly Catholic, theologians.² This philosophical period culminated with the 1887 *Mitteilungen zur Lehre vom Transfiniten* (*Communications on the Theory of the Transfinite*).

Before looking more in depth at this work, it is important to see how Cantor was reacting to the cultural and political evolution around him. We will use some of his correspondence to exemplify it.

The cultural milieu

Cantor wrote to and about a certain Julius Langbehn, at the time the famous author of the book *Rembrandt als Erzieher des deutschen Volkes* (*Rembrandt as Educator of the German People*). Langbehn represented the growing Romantic reaction to the perceived danger of industrialization and mechanization. In his case, this was dangerously coupled with Nietzscheanism and pan-Germanism. Everything “bad”



Dino de Paoli at a conference in 1993. " 'Cantorian mathematicians' tend not only to forget the background of Cantor's discoveries, but also to oppose the philosophy which led to such breakthroughs."

was "un-German," coming either from "outside"—specifically, France—or from "internal enemies," who soon will be called the "Semites" and the "Papists." In this fake opposition to mechanization, German culture is reduced to the status of "defender of nature."

Langbehn, for example, writes: "Zola wants to replace organicism with mechanics; that is French, and that is un-German." And worse: "The German thus already dominates Europe, as an aristocrat; and, as a democrat, he also dominates America; it will probably not take long until he dominates the world, as Man."

Cantor, although at the beginning attracted by Langbehn, was not fooled, and his reaction shows some interesting political insight. He wrote to a Jesuit, Father Baumgartner, on May 20, 1891:

"What was your impression of *Rembrandt as Educator of the German People*? I see it as nothing but one of many attempts to turn all Germans into Bismarcks behind their backs. For Rembrandt here is nothing but a mask for Bismarck."

Baumgartner himself later wrote an article attacking Langbehn's book, and to this Cantor commented on May 25, 1891:

"I entirely agree with your review of Langbehn's book. . . . I do not think it impossible, that Langbehn will want to speak with you, perhaps already this summer; then let us keep it to ourselves, that we have corresponded about this queer fellow."

Cantor explained how he met Langbehn, and added:

"He no longer has relatives, and he is unmarried. I asked

him about writers to whom he felt closest, he named Hölderlin, Novalis (fragments from the posthumous writings), Rahel (a book of memoirs), De Lagarde (German writings),³ Grabbe, and particularly Nietzsche. He visited the latter in the psychiatric sanatorium one year ago, lived with him there for eight days." (P 200)

On May 26, 1891 Cantor wrote to Langbehn:

"To me, in any case, you seem to be more competent in matters of art, than in other areas. I cannot close my eyes, for example, to the mistakes in the book demonstrated in politics by the course of developments in Germany [the fall of Bismarck], nor does his 'strange' political tendency appeal to me as a practical Hohenzollern. Furthermore, since you expressly ask me for the truth, I am obliged to tell you, that I, as a positive Christian, cannot condone the view of Christianity you present, and that I deem various passages to be outright blasphemous." (P 99)

Cantor also perceived and fought the growing anti-Semitism. On Aug. 26, 1891, he wrote about "Bismarck-allied, anti-Semitic, so-called German National Students." And more specifically:

"This is a sad, crude, empty-phraseology group of heros with only big mouths, which I know quite well from here in Halle, where they have a branch of a club called 'Association of German Students'. . . . In the most recent history of these people . . . I am libeled as an 'advocate of battle for the Jews,' because I have politely but emphatically and honestly rejected this anti-Semitism of children and Protestant clerics, which is of no benefit, but only damage to the German people. I was attacked a great deal in anti-Semitic newspapers

nine years ago.” (P 100)

Or, on Feb. 4, 1896, in a letter to F. Heiner concerning the refusal to employ philosopher Edmund Husserl in Freiburg:

“If he has no other and no better reason to be reserved with respect to Mr. Husserl, than that he . . . revealed, that he [Husserl] came from Judaism and became a Christian, then here once more I see the sad case, one which unfortunately happens frequently in Germany, of a Catholic Christian who most flagrantly renounces the most sacred tradition of the church! Shall I recall that, apart from the most sacred person of our Savior, his 12 apostles were all Jews? Does the most esteemed colleague not know, that the church had always commanded, that the Jewish question be seen only as a religious issue? Does he perhaps believe, that the poison of racial anti-Semitism appearing among so many German Catholics, has ever been sanctioned by the church? Or does he perhaps think that, at this moment, under the most glorious papacy of Leo XIII, racial anti-Semitism will perhaps be tolerated in Rome? Has he perhaps not been informed about Archbishop Kohn, residing in Austria, who is the son of an Israelite bourgeois? Has he never heard the name of the honorable priest from Alsace, Ratisbonne? Has he still not received the henotic encyclicals of the great Leo of June 20, 1894 and April 14, 1905? Does he perhaps believe, that he will be able to support the great work of unification, which His Holiness has embarked upon in the last years of his life, by promoting such hate-filled anti-Christian racism? It is not impossible that I will have the opportunity over the Easter holidays in Rome, to put the material into the proper hands, which I have collected for years on this issue of Judaism, which has been provided me, however, in only the rarest cases by Catholic clerics.” (M 379)

Cantor’s morality, religiosity, and epistemology led to the scientific breakthrough which gave him the key to grasp the similarities and consequences of the apparently opposite views of Nietzsche and naturalism on the one side, and the logic of Bertrand Russell and Gottlob Frege on the other, although his reaction seems stronger against the first.

On Feb. 24, 1900, he wrote to F. Loofs:

“Dear colleague, many thanks for your ‘Anti-Haeckel,’ I think it is very valuable, that the presumptuous veil of science be torn off, for the benefit of broader circles, from Haeckel’s shameless assaults against Christianity. The great diffidence in engaging in forthright polemics (so prevalent in our circles!) must be given up in the face of such good-for-nothings. Hopefully co-combatants will join you, so that it will not be necessary for you to take up the matter again personally! By the way, I recently had the opportunity to acquire a better picture of so-called Nietzschean philosophy (an appendage of Haeckel’s monist philosophy of development). Among us, on account of the stylistic charm, you will find uncritical *recognition*, which, in view of the perverse content and herostratic-anti-Christian motivation, appears to

me *most alarming*. The need for novelty and the plethora of the philosophical structure makes our philosophers morally blind and rashly willing to include anyone in their historical studies, who turns up claiming he has a new system. Thus does the ambitious innovator always achieve his goal; he becomes a famous philosopher, and the youth is corrupted in grand style.” (m appendix)

Cantor’s reaction against Ernst Haeckel (1834-1919) is very relevant. Haeckel, the main German Darwinist, was also the founder of a so-called scientific “religion” which is nothing but a pagan cult of Mother Earth. He tries to give himself a cheap cover as a progressive by adopting “anti-Papist” rhetoric; for example, he is sometimes presented as a “modern defender of women,” while in reality defending the “female” as a symbol of the natural religion, against the “male” of Judeo-Christianity. His “Monistenbund” propagated the thesis that evolution proves scientifically the equality between inorganic and organic, life and conscience. Given this *simple continuous evolution*, human beings are nothing special; on the contrary, certain people and “races” are even inferior to certain animals. This is brutally expressed also by one of Haeckel’s references, to E. Renan, who, in his 1863 *Life of Jesus*, has a long tirade against the Semitic “races,” and declares that Darwinian selection should allow the creation of a superior race. Haeckel’s “religion” was obviously very much appreciated by the Nazis: In January 1939, the E. Haeckel House was given the title of “Institute for the History of Zoology, in Particular the Theory of Development,” with the approval of the Reich Minister for Science, Education, and Public Schools. The chairman of the E. Haeckel Society in Jena, Prof. Dr. V. Franz, published, with the obvious authorizations in 1943, a biography of Haeckel which ended with a quotation from Hitler.

Although less well known, on the other side of the opposition to Cantor we find the “logical determinism” of B. Russell and G. Frege. They are also “monists,” although instead of “nature,” they use logic to deny the existence of anything special in man. This is expressed with the attempt to completely formalize human thinking so as to reduce it to a simple mechanism. Also this “absolute science,” like that of Nietzsche and Haeckel, needed as its premise a vicious attack against Christianity. Russell’s outbursts are very well known; less well known is that in Germany, Frege, then professor at Jena, accompanied his rhetorical outbursts against the “ultramontanists” with an endorsement of Hitler, and even with anticipating some of Hitler’s measures. This is clearly expressed in Frege’s 1924 *Tagebuch*, of which I choose only one passage:

“One can recognize that there are quite respectable Jews, and still think it a misfortune that there are so many Jews in Germany, and that they have full political equality with citizens of Aryan descent; but how little has the wish been fulfilled, that Jews in Germany lose their political rights, or better yet, disappear from Germany . . . but how can one

reliably distinguish Jews from non-Jews? That might have been relatively easy 60 years ago. To me, it seems difficult today.”⁴ In 1935, ten years later, Hitler will find the way to solve Frege’s logical dilemma.

Cantor’s contribution

Cantor’s name is associated with the discovery of a method to give a number to infinite processes, a number which he called “transfinite.” This was not deduced from known mathematical axioms; instead, it required the introduction of new concepts and hypotheses. Although it was new in its use in mathematics, “transfiniteness” had previously more or less implicitly shaped the debates in theology, philosophy, and art, under the name of “actual infinity.” It is in this broader sense, that we will briefly present it here.

To assign a “number,” that is, a constant, to an infinite process, means, as one can see, to use two terms in apparent contradiction. The same type of paradoxical expressions are to be found in all recorded forms of culture, although under many different metaphors. For example, under the relations: unity-multiplicity, being-becoming, limited-unlimited, subject-infinite predicates, etc. This universality of use indicates indeed the ontological, existential importance, but also the difficulty of verbal expression, of our active form of knowing, of our creative powers.

What is relevant for us here, is the kind of answer which has been given to this “paradox,” and what kind of cultural milieu has been shaped around it. Let us see two illustrations directly linked to Cantor.

1) Plato realized that the apparent paradox resulted only from the privilege which had been given to sense-perception in our process of knowledge and change of an environment which *includes* ourself. Sense-perception cannot “see” the mental power which determines our and others’ actions; it can see only its results or “predicates.” This is the general mistake of simple monism, which de facto reduces the human being to a mere object of the observed “nature,” eliminating any free will or creative power in the individual. Plato asserted, against the monists, the *higher power*, first, of the “ideas” over the ordered changes of predicates, and then of the Subject over the ordered changes of ideas. In so doing, he solved the paradox without eliminating one of the terms, without calling the unity, the subject, or the action of change, “a simple illusion.” The further step of Plato, whose importance will become clearer when linked with the so-called “Cantor antinomies” or the “paradox of the Absolute,” was its formulation against the sophists: “Man cannot be the complete measure of everything, but God is so.”

Nearer to Cantor’s period, we see the contrary approach:

2a) Haeckel’s manifesto in his *Principles of Theophysics*:

“1) God is nature itself, eternal and indestructible. 2) God himself is unaware of the laws of nature, immutable: *Universum perpetuum mobile*. 3) God has no free will and no whim. . . . 6) God as blind fate (*fatum*) is the universal.”

2b) Russell, more formal, and similar to Frege:

“It was assumed as axiomatic that the class as one is to be found wherever there is a class as many . . . this need not to be universally admitted, and appears to have been the source of the contradiction. By denying it, therefore, the whole difficulty will be overcome.” (In other words, in the face of the paradox of the One and the Many, we will eliminate the “One.”) This philosophical difference reflects, in reality, two different ways of considering the role of man in society. For Plato, man has something “higher” and sacred; for Haeckel and company, man is, and can be used as, a mere beast. Cantor, in the foundation of his transfinite, goes explicitly back to Plato. The following is, for example, his formulation:

“By ‘manifold’ or ‘aggregate,’ I generally understand every multiplicity which can be thought of as one, i.e., any totality [*Inbegriff*] of definite elements which by means of a law can be bound up into a whole, and I believe that in this I am defining something which is related to the Platonic *eidōs* or *idea*, as well as to that which Plato calls *mixon* in his dialogue ‘Philebus, or the Highest Good.’ ” (W 204)

The unity of an infinite Many is the Transfinite or “bounded infinity.” The specific necessity to use infinite processes, forced itself through human history against all attempts to avoid it. It appears mainly when we are relating elements or modes or processes whose evolution, or whose power, is of a different order. That is, it expresses incompleteness in the type of measurement we use, or what is called incommensurability. For example, the famous impossibility to completely measure a circle with a line. To put oneself, in a sense, “above” infinity, to “bound” it, one has to grasp that:

1) Certain processes of evolution present a fundamental discontinuity between levels or modes or types, making it impossible to relate them with the simple concept of “equality.” Man is not equal to a beast. Different moments of human history and life are incommensurably different if *divided* by a creative thought and action.

2) Gaps seem to separate such moments or elements, gaps which are infinite if one measures one element with the other. Nevertheless, we know there is some “reason” for the gap. A creative *idea* has shaped such incommensurable changes. The key of Platonic philosophy is precisely to make clear the ontological and primary role of such “ideas” over the apparently infinite “more or less” of measures and definitions. Creative ideas *reflect* the higher power of the author or subject, and so they can have the function of “bounding” and of giving “harmony” to the otherwise mysterious infinite gaps.

This is usually clearly expressed in art. But it shaped also the evolution of mathematics, and I will here indicate few examples:

1) In projective geometry (perspective), it appears around the notion of point at infinity or focal point, the point which bounds and defines the harmony of the plane.

2) In the concept of "limit" and "irrational number," for example, here is how Leibniz introduces it:

"If we consider the totality of a series of numbers, then we are able to comprehend it as a totality, even if it is infinite, insofar as it is determined in its progression by some law. Its totality, of course, is not expressible by a simple number."

Leibniz here considers only ordered processes which seem to "converge" toward some "limit." Limit, like the previous "point at infinity," has a different and higher order of "existence" than the ordered process. One can grasp this concept better in Leibniz's *Monadology*. Cantor will generalize this to any type of ordered change, including what appears as "divergent," that is, when the "limit" seems completely hidden and invisible. For example, our normal way of "counting" seems to be a "limitless" process:

1, 2, 3, 4, . . .

If we indicate this process with *I*, then Cantor says it is possible also here "to think a *new* number, which is to be the expression for the [fact] that the whole *Inbegriff* (*I*) is given in its natural succession according to the law."

The bounding idea of this type of general process is called by Cantor "transfinite number," and what we are actually doing, is to find causality or "*reason*," or continuity, where it is not immediately visible. But we establish continuity without *negating* the discontinuity, the change, ordered through a "creative idea."

Let us see how this is expressed again by Leibniz:

"The principle of Continuity means that no sudden vanishing happens without us being able to determine the *reason* for it in the form of point of inflections, singularities, etc." (Letter to Varignon, 1702)

"In a geometric line, there are special points of singularity, and as there are lines which have an *infinite* number of such points, we must in like manner conceive in the person's life, periods of extraordinary changes which are not outside general law." (Letter to De Montmort, 1715)

Or by Bernhard Riemann (a great mathematician who here is using Herbart's philosophy):

"Once the concept of self-subsisting things has been formed, upon reflection on the fact that things change, which *contradicts* the concept of self-subsisting things, the task arises to maintain this already proven conception as far as possible. Out of this there emerge at the same time the concept of continuous change and the concept of causality. What is observed is only the transition of a thing from one condition into another, or to speak more generally, from a certain manner of determination into another, without a leap being perceived."

In the case of Cantor, I want to show this also in a specific, applied form. He used this concept against Newtonian mechanics, where causality is reduced to simple determinism by limiting the possibility of changes or motions to continuous simple space. Cantor instead proves that:

"The *hypothesis of the continuity of space* is thus nothing

but a per se arbitrary assumption of complete, one-to-one correspondence between the three-dimensional purely arithmetic continuum (x, y, z), and the space underlying the world of phenomena. But our thinking can with equal ease abstract quite well from single space-points, even if they are everywhere dense, and it can form the concept of a *discontinuous* three-dimensional space (T) of the previously characterized Type. The question which then poses itself, whether also in such *discontinuous* spaces (T) *continuous movement* can be thought, must, according to what has been said previously, be unconditionally *affirmed*. . . . The attempt thus suggests itself, to undertake a modified valid mechanics for spaces of the constitution (T)." (W 156)

In a letter of Oct. 19, 1886, he then split the notion of motion from the one of speed. That is, we can have changes which nevertheless are not measurable linearly:

"To state my opinion more clearly, I note: 'direction' and 'velocity' are, in my estimation, only accidents of movement, i.e., they *may* also be lacking, they are not fundamental for a general concept of movement. I think that it is *impossible* to prove, that every movement must necessarily be attended by 'velocity' and 'direction.' Movement with those accidents is, upon precise observation, just as invisible and, if you will, just as dark, as movement without them." (P 110)

Without entering into difficult details, it should be noted that this is a clear anticipation of the type of "mechanics of the transitions" which had its first partial realization with the concepts of atomic "quantum of action" and "Brownian movements."

1895: the Aleph series and the antinomies

Cantor's breakthrough was based not simply on the generalization of a concept such as "bounded infinity," but also, and especially, on finding a new method of measuring such infinities. In this way, he could find common properties and differences not otherwise visible with the simple use of the "greater than or less than." He derived this new approach explicitly from projective geometry, and he called the common property of ordered infinities "*Mächtigkeit*" [power], or "cardinal transfinite," or later, "Aleph" (\aleph). To his astonishment, he found out that not all the ordinal transfinites could be measured by the same *Mächtigkeit*; some were "not denumerable" or, again, incommensurable. For the simple field of "arithmetic," already he needed at least 2 Alephs, and there was no apparent reason to stop at 2. This necessary passage from one to the other through incommensurability, this generation of an "Aleph series," is nothing but a higher form of what we called before a discontinuous continuity. Or, using Leibniz's language, one seems to need different orders and different types of orders of the notion of "continuity" or "reason." Plato himself had clearly said in "The Republic" that "ideas" themselves change, bounded by *higher* ones. Real discoveries, says Plato, are not derived by deductions from given axioms, but by realizing that axioms are

“hypothesis,” and discoveries result in being able to change them in theorems of “higher hypothesis.”

It is an impossible thought, for most people to live with the subjective experience of the “lacking of something.” Most of us want a final proof, want to avoid the Socratic “I know that I do not know.” We can live in our world as individuals only by new theories, new ideas of transformations, new discovery of laws. Couldn’t we also have a measurable “idea” of the world in its complete totality and unity? Cantor, especially as a former admirer of Spinoza, also had to face this problem again and again, until he abandoned Spinoza for Leibniz and Plato. So he asked himself the obvious question: We have now an ordered generation of Alephs, which represents a world of creative changes; is there, then, a possible last or maximum Aleph? Maximum here means the final unity of all the preceding ones:

[$\aleph_1, \aleph_2, \aleph_3, \dots$ etc.]

Or in other words: 1) Is there a final theory of the “universe,” where creative individuals can still live? 2) Is there a complete knowledge of ourselves, which could be reproduced in a machine? 3) Is there an absolute unchangeable something in physical nature?

Cantor’s negative answer came out loudly and clearly:

“The totality of all Alephs is such, as cannot be comprehended as a determinate well-defined *finished* manifold. Were this the case, a determinate Aleph would follow upon this totality according to its magnitude, which would thus both belong to this totality (as an element), and also not belong to it, which would be a *contradiction* . . . totalities which we cannot comprehend as manifolds (one example of which is the totality of all Alephs, as proven above), are what I called ‘*absolute infinite*’ totalities years ago, and distinguished them strictly from transfinite manifolds.”

In brief: The world in which we live cannot become our Idea; we did not create it.

The discovery of this paradox completely upset the “imperial monism” of G. Frege and B. Russell. Their attempt to reduce human thinking to a mechanized process was proved to be impossible, first by Cantor and later more precisely by Kurt Gödel. Russell tried to escape the problem, as we saw before, by rejoining Nietzsche. There is no “unity,” no “universal,” no certitude or truth or morality or Absolute at all. In other words, as Nietzsche said, “God is dead.”

If Haeckel had spent more time thinking, he also would have realized the impossible paradox of his “monism”: unchanging nature. His god was as dead as that of Nietzsche, and what was left was the evil justification of the “natural” predetermined superiority of the right of the strongest. Cantor, as we have seen, clearly realized the Nietzsche-Haeckel similitude.

Naturally, a political, dictatorial system can always solve the paradox: Simply eliminate the inner reference and effects of the power of human creativity: the lawful changing of nature. Physicists pretend they are immune to this paradox,

because they consider this to be “subjective,” while they are handling the “objective” world. They do not see that the paradox appears precisely in the *assumptions* of their “theory” of the world.

Let us take the example of Newton, another self-defined “unitarian.” He introduced something “unchangeable” and “objective” in the world, but pretended he had found it there: absolute space and time. It was Leibniz who first reacted and attacked this: “The *concept* of space is *relative*, it is a concept of the order of co-existences, while time is a *concept* of the order of succession.”

And Cantor: “Time, in my view, is nothing but an auxiliary and relational concept, by which the relations between different movements which occur in nature and are observed by us, are ascertained. Such a thing as *objective* or *absolute time* occurs nowhere in nature.” (W 191) The same is true for space, adds Cantor. Plato had expressed it more poetically in the “Timaeus”: Time and space are created by God.

In the “imperial” monism, the law of nature becomes reduced to “*vita mea mors tua*” (“my life is your death”). If that is “nature,” then we know that we are able to transcend nature. We do it by *increasing* the carrying “power” of nature through science and technology, and so eliminating the justification for the competition for the need of more “living space.” Nature herself becomes less brutal in that way.

Cantor’s “paradox” of the “absolute infinite,” was for him a discovery of a characteristic of our human world. For Plato, as for Nicolaus of Cusa, Leibniz, and Cantor, this paradox, this realization of the existence of the ordered transcending power which guarantees the existence of a free and not arbitrary will, was the “*reflection*” of the Absolute. The world is not our idea, but it is an Idea.

“For now we see through a glass, darkly; but then face to face. . . . And now abideth faith, hope, charity, these three; but the greatest of these is charity.” (1 Cor. 13:12-13)

Notes

1. Sources of quotations from Cantor’s works and letters are designated by the following abbreviations:

M plus page number refers to H. Męnschkowski and Winfried Nilson (ed.), *Georg Cantor Briefe* (Berlin: Springer-Verlag, 1991).

m plus page number refers to H. Męnschkowski, *Werk und Leben G. Cantors* (Braunschweig: Vieweg, 1967).

P plus page number refers to W. Purkert and H.J. Ilgands, *G. Cantor 1848-1918* (Birkhäuser Verlag, 1987).

W plus page number refers to *Cantors Werke*.

2. For an English translation of the correspondence between Cantor and Cardinal Franzelin, see “On the Theory of the Transfinite,” *Fidelio*, Fall 1994.

3. Paul Anton de Lagarde (1827-91) was one among many “Orientalists.” He is presented thus in the German Brockhaus encyclopedia: “The German nations can only be united by a national Christianity. Lagarde combined this demand with a fierce critique of Christianity and the church of his time, of Judaism, especially St. Paul. . . . under National Socialism, Lagarde stood in high esteem (A. Rosenberg).”

4. See G. Frege, “Tagebuch,” in *Deutsche Zeitschrift für Philosophie*, Berlin 42 (1994), 5, pp. 1067-1098.