## **EIRScience & Technology**

# The human mind is not a computer

Aristotle tried to reduce man's thinking to either/or propositions, but a succession of great thinkers from Plato, to Cusa, to Cantor proved him wrong. Laurence Hecht reports on a speech by Dino de Paoli.

Dino de Paoli, author of "Georg Cantor's Contribution to the Study of Human Mind—A Refutation of Artificial Intelligence," summarized the concept of the transfinite in a rigorous, nearly two-hour-long presentation to the U.S. conference of the International Caucus of Labor Committees and Schiller Institute, in Alexandria, Virginia over the 1991 Labor Day weekend. The following is a report on his speech, which originally included many more graphics than we are able to reproduce here. The report was prepared by Laurence Hecht. Readers interested in pursuing ideas further may wish to consult De Paoli's study of the 19th-century mathematician and philosopher Georg Cantor (1845-1918), which appeared in the Summer 1991 issue of 21st Century Science & Technology.

At the outset of his speech, Dino de Paoli posed the fundamental point at issue as being not a formal question in mathematics, but, rather, the same question that divides the world view of Socrates and Plato from Aristotle—whether human creativity exists as a force that changes the universe, and whether man can know the truth in connection with the Absolute. Thus the debate today on the fundamentals of science is, "our debate, our issue." It is around what is creativity, how does it function. The technical terms of the debate: Singularities, discontinuities, phase changes, big bang theories, etc., are but different faces of the same issue, LaRouche's issue: can the physical universe be intelligibly represented by man? The answer will be yes, but for creativity not to be viewed as a mystic occult power, it demands that the "geometry" of the universe be "curved."

Aristotelianism, he said, is dualism. You are forced to make a choice between one of the two: the one or the many,

spirit or matter, logic or intuition, the square or the circle. "When faced with such a duality of choices," he warned, "always pick the third."

The Aristotelian view is reflected in Euclid's mode of presentation of geometry. The system attempts to present itself as a logically complete set of axiomatic rules of construction. But one need go no further than the first proposition of the first book of Euclid's *Elements* to see that the consistency breaks down. The problem is that the system requires three essential postulates:

- 1) There is a line
- 2) There is an angle.
- 3) There is a circle with its center.

Why so? Why is it necessary to have an angle as postulate?

Because the real underlying assumption of Euclid's system is of *linearity*. But, the construction of even a single angle requires *rotation* of the line, and so implies a circle! Thus the universe cannot be built up of merely points and lines, merely linear measure. This formalism requires also the assumption of another entity in the universe: the circle, the One, the unlimited.

Now the problem that arises from a formal standpoint, is that once this entity, the circle, is allowed in, as it must be, it is not possible to find a common measure between the circle and the line (or square) without allowing in infinities. (For example, what we know today as "pi," ( $\pi$ ) the ratio of the circle's diameter to circumference, can only be formally represented as an infinite series; it cannot be expressed as a simple integer or ratio of integers that correspond to constructible distances on a line.) And if the infinite is allowed in, then other paradoxes arise—Zeno's paradoxes and so



The master Plato (left) and his recalcitrant student, Aristotle, walk into this busy scene of learning in a detail from Raphael's 1508 fresco, The School of Athens. The two figures appear to be stepping into the scene directly out of a beautiful cloud-filled sky, which we see framed by two curved arches, perhaps a visual metaphor for the studies of curved space which the artist and his predecessor, Leonardo, had carried out.

forth—in such a formal system. The Aristotelians will allow the infinite in as a necessary adjustment to their system, but they must ban it from exerting any real action on the world. Thus there is no possible way to create anything new in their system; we can only rearrange the already-existing things.

#### How the oligarchy defines your options

The entire scientific, epistemological, and theological debate over the past 2,500 years revolves around this question. It revolves around the relation between the linear and the curved: the problem of the squaring of the circle. We can schematize the debate approximately as follows:

A stands for the discrete, the linear, the many, body, matter, *object*.

B stands for the the continuum, curved, one, soul, force, subject.

Since A and B are incommensurable with each other, you are given the following choices:

• First option: Choose *B*, in itself, the One. This is holism, Oriental mysticism, German Idealism, religious fundamentalism, chaos theories, Romanticism, and the New Age.

• Second option: Choose A, in itself, the Many. This leads to pluralism, Illuminism, logicism, artificial intelligence, secular humanism, Marxism, mechanism.

• Third option: Choose the simple sum of A and B. You

may allow that both exist, but you view B as unmeasurable, unintelligible, only a symbol. This is the choice of theological Aristotelianism, Kantianism, and Cartesianism.

• The Socratic third way: The Socratic third way, none of the above options, is what we choose. Eudoxus (see Appendix), a student and very close follower of Plato, tried to develop a mathematical process of infinite approximation between A (the polygon) and B (the circle), to find a common measure among "incommensurables." This included a process of approximation or exhaustion for finding the area of the circle. Despite its limitations, its implicit assumption is intelligibility in the universe, and it is the basis for the reason western culture developed technology.

Using a projector to show an illustration of a circle containing an inscribed polygon, De Paoli described the process of approximating the immeasurable area of the circle by finding the measurable area of successive polygons, each one with more sides and therefore more like the circle. However, no polygon, no matter how many-sided, ever equals the circle. The key to the Socratic method of exhaustion is the assumption of intelligibility in the universe, recognizing that any attempt at portrayal is only approximate and will ultimately be superseded, but that each approximation, worked out as an algorithm, i.e. a precise law or set of laws, a One, defines one economic-technological space—to use LaRouche's image.

Plato had already described the resolution of this apparent paradox given by the dualism of the One (the circle) and the Many (the polygon) in the dialogue Philebus, or the *Highest Good.* There, Socrates, after having described "his method," says that there are not two but three principles of construction, and a fourth which is the cause of them.

a) the simple, unlimited many; the indefinite more or less

b) the limit—the simple oneness

c) the third kind, the mixton, which, in his words, is "the coming into being resulting from those measures which are achieved with the aid of the limit," or "the being that has come to be the mixture of these two," i.e. (a) and (b)

d) the sufficient reason, of which he says, "All things which come to be should come to be because of some cause . . . and a cause is a maker," or "the cause of the mixture and of the coming-to-be" (Philebus, Sections 26, 27 a, b, c, d).

We can in fact reduce this to three principles:

A) The linear, limited, or the unlimited series of created ones—Object(s)

B) The creating of the created ones—Verb

C) The Sufficient Reason (logos); the cause or the maker-Subject.

Or, reduce it even further to:

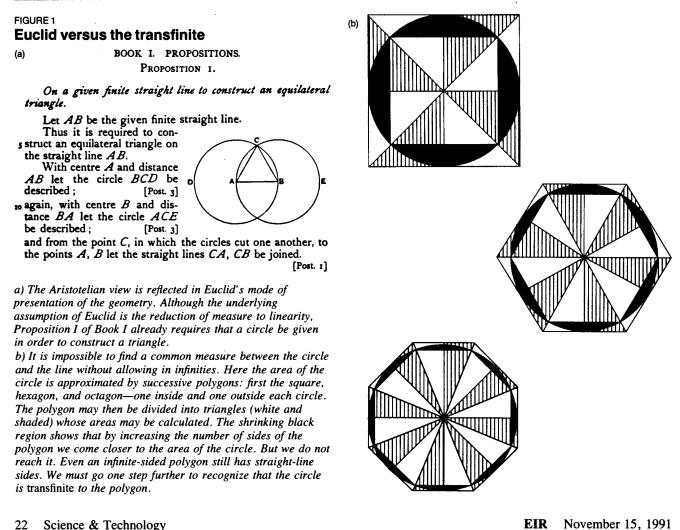
A) the created ones

B) the creating one, and

C) the Reason of the One Creator.

The key is that it is only the *unity* of the three principles which is the real characteristic of existence and intelligible transformation. This is the matrix of the western Socratic-Christian tendency in science, philosophy, theology, and so forth.

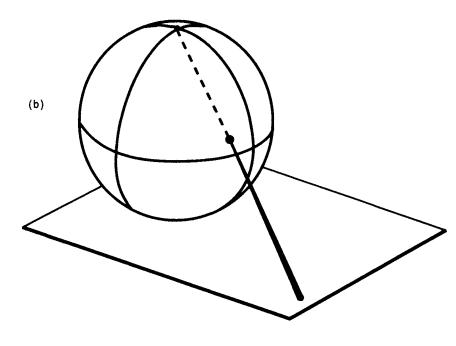
To illustrate. De Paoli went back to the Archimedean construction for the determination of the area of the circle by finding the limit of areas of the inscribing and circumscribing polygons. Archimedes did not explain it clearly, he said, but if the process is understood as Plato understood it, it is a correct "approximation." Archimedes also shows that by



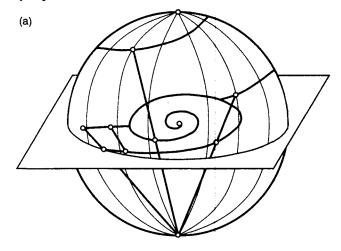
using circular and linear actions, we generate curves (actions), which would be impossible under the strict rules of dualism separating line from circle. He sees the spiral, for example, as both a rotation and a linear translation at the same time. But in reality, De Paoli says, it is a doubly curved action. We see this when we look at the spiral projected on the sphere (loxodrome curve). It has no linear component, but is in reality doubly curved. Curved action, De Paoli says, defines all Euclidean constructions, and doubly curved action defines what is not constructible in the Euclidean system.

#### **Renaissance improvements on Archimedes**

Archimedes' work was brought into the modern world through translations in approximately A.D. 1150 in Moorish Spain. From there it was spread into Italy and France. Nicolaus of Cusa made a crucial improvement in explicitly recognizing the constant value of curvature, and that even an infinite polygon *is not self-bounded*, but is bounded by the circle which is *of a different nature from the polygon*. The circle, Cusa recognizes, must be defined as isoperimetric action (the

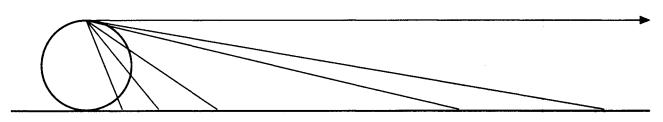


#### FIGURE 2 The loxodrome and the stereographic projection

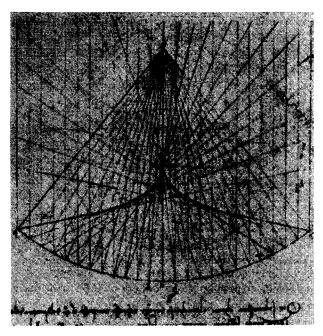


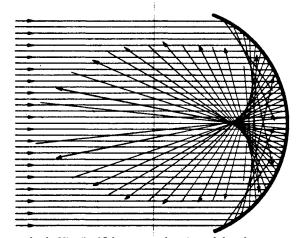
a) The loxodrome is the curve traced on the surface of the Earth by a ship maintaining a constant compass bearing, such as due northeast. The diagram shows a loxodrome traced on the upper half of a sphere, and its projection onto a plane slicing the sphere through the equator. The projected plane curve is known as the logarithmic spiral. The point of projection is the south pole. Note that the loxodrome is doubly curved; it will not sit on a plane.

b) The stereographic projection maps any point (P) of the plane to a unique point (P') on the sphere. The point P' is determined by drawing a line connecting P to the north pole of the sphere. Where this line intersects the sphere's surface is the point P'. As we move farther out on the plane, the line connecting P to the pole begins to become parallel to the plane, and intersects the sphere higher into the northern latitudes, as the side (cross-sectional) view illustrates. Finally, the infinitely distant point on the plane is represented by the north pole.



#### FIGURE 3 The caustic





Leonardo da Vinci's 15th-century drawing of the phenomenon known today as "spherical aberration," and a modern illustration (right). The light reflecting from a spherical mirror, or bent (refracted) through a spherically curved lens does not focus at a single point, but spreads out over an area (the cuspshaped curve in both figures), known as the "caustic," or burning curve. The term dates from Archimedes' employment of large mirrors as instruments of warfare.

Source for right-hand illustration: Francis W. Sears, Optics, © 1949, by Addison-Wesley Co., Inc. Reprinted with permission of the publisher.

#### mixton of Plato).

Leonardo da Vinci then introduced another crucial factor in the relation between the curved and the linear. He demonstrated that the incommensurability is expressed geometrically in the caustic. That is, light reflected or refracted through most curved surfaces will not focus at a point, but spreads the focus out into a curve generically known as a caustic.

The stereographic projection shows that the circle is topologically similar to the plane, *if we include the point at infinity*, that is, that it is possible to map every point of an infinitely extended plane onto a finite sphere, by making the north pole of the sphere correspond "to the infinite point(s)" on the plane. This introduction of the point at infinity is fundamentally non-Aristotelian and is also crucial to the method of linear perspective. But there is still a crucial difference between linear and curved (or curvilinear) perspective, De Paoli said.

He illustrated this with a slide showing a cross-sectional slice of a stereographic projection Even if each point of the line can be uniquely mapped to a particular point on the circle, the metric of the projection is different, and this shows up in the necessity for the formation of a caustic in projections involving curved surfaces. The implications of this for Cantor show up in the different types of ordinal numbers that can exist.

Gottfried Leibniz (1646-1716) continues the process of

development of this concept. His point of reference is not strictly or merely squaring of the circle. He considers *action*, changes, and the "integration" (or "measure") of such action—that is, its intelligibility.

In a 1702 letter to Varignon, he wrote:

"Physics is based on the sufficient reason, while geometry, which is its representation, is based on the principle of Continuity, which means that no sudden vanishing happens without our being able to determine the *reason* for it in the form of points of inflection, singularities, etc., so that some mathematical expression will be created to include such singularities in nature so as to avoid the inclusion of chance or miracles."

Then in a letter to R. de Montmort in 1715, he wrote:

"Now as in a geometric line there are certain special points of singularities, etc., and as there are lines which have an *infinite* number of such points, we must in like manner conceive in the animal's or person's life, periods of extraordinary changes which are not outside general law, just as the special points on a curve may be determined by its general nature or its equation."

To understand what he means by such "special points," by "integration," and so forth, you must not look at the squaring of the circle in a purely formalistic, or Aristotelian way. You must think back to Plato's three principles, De Paoli said. Think back to the continuous action that lies in between the different segments of the line, in between the distinct created "ones." Think of LaRouche's approach and then come back to Leibniz. Think in terms of action.

Now if you think of the polygon as line segments connected by angles, ask what actually are such angles? Why did Euclid have to introduce as postulates, the circle, the line, and an angle? The angles are what today would be called a first species discontinuity—a non-differentiable, non-linearizable part of a function. But they are the result in A of something happening in B because of C—harmonic circular action. The simple figure of a triangle inscribed in a circle illustrates the point:

The triangle exists only because a linear action is forced by the circle to bend, to form angles! This is what Leibniz was speaking about.

#### **Cantor's corrections of formal mathematics**

The misunderstanding that arose in formal mathematics was the error of considering as "measurable" only the line segment, and *not the discontinuities*. So all of classical integration is an attempt to go as deeply as possible into the "small" to find areas without the "angle," and then to do the sum, in order to obtain a smooth, continuous line, a so-called "rectified curve." That is uniform linear action. Thus, not only do we avoid the circular action. We even linearize the triangle, reducing it to points, and then do the sum. Yet, the angles are an expression of the circle, which is what we were supposed to find in the first place!

So Leibniz was the first to indicate, in the specific area of mathematics, the importance of such discontinuities. This led directly to Fourier's finite discontinuites of first species, then to Dirichlet, and then to the Riemann integral. The Riemann integral was the resolution of physical problems even in the presence of infinite discontinuities, but finite in finite intervals and of the first species (angles). In other words, it made it possible to handle physical problems which are described as an infinite polygon, but which still have a line, or *side*, in small finite intervals.

But what about when the approximation moves closer toward the circle, and the angles tend more to be really infinite in number, while the line (side) tends toward zero? Then the mathematical image becomes not yet the circle, but *a dense set of points circularly distributed*. Between the points there are still "holes," "infinities" again. The angle discontinuity, which is algebraic, is now coupled with "holes," a "transcendental" discontinuity, and the more you go into the small, the more such transcendentals you will find.

This brings the matter to Georg Cantor, De Paoli said, referencing his article in 21st Century Science & Technology (Summer 1991). Cantor makes infinity into an intelligible issue. And, he clarified more precisely what we characterized above as the A and B.

A is proved to be a geometry of zero curvature, that is, linear area. This describes whatever new level of algebraic

algorithm ("deductive lattice") we have already created the measurable, denumerable, classically integrable. He also proved that *this is not a closed area*, *but is unlimited, has no maximum*. To find closed segments here, that is, harmonically distributed closed actions, requires "limits" which lie somewhere else, they are "transcendental." In short, A is the "denumerable" area, in Cantorian language.

B is now proved to include all the numbers, geometric figures, and functions. B is not *integrable*, not measurable with an A type of metric. B is proved to be *non-linear*. It is positively curved, and includes all of A plus the point at infinity, like the sphere. The transcendental numbers define its point of separation from A.

So, in simple words, Cantor proves that the circle is not squareable, because of the existence of curvature at any and each infinitesimal part expressed by the existence of an infinity of transcendental numbers, infinite in any finite interval.

But is the square curvable?

Cantor tried to find and to specify some relation between the A and B elaborating on the principle of Leibnizian continuity or intelligibility. He tried the usual way of examining series which go to infinity, but with no success. So he makes the conceptual jump, and introduces a transfinite relation. To understand this, think of Plato's mixton-the coming into being. The circle-or better, the curved action-is the unitary measure, but is also the "coming into being of unities." Vis-à-vis the polygons, the circle is an actual infinite polygon, that is, it completely represents their construction as a totality. It is transcendental to any one of them or to any linear combination of them, but still causing its existence as an ordered harmonic set of segments connected by angles, as we showed. The existence and distributions of the angles are completely determined by the existence of curved action, not by the polygons.

#### **Transfinite numbers**

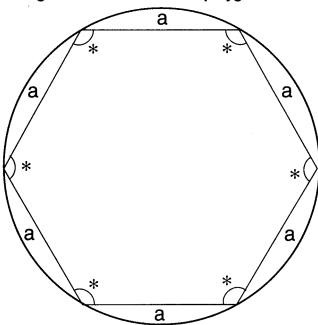
De Paoli concluded with a more formal description of the Cantorian transfinite numbers, pedagogically elucidated by his earlier image of the growing polygon in the circle, and by his clearly delineated distinction of the two categories A and B.

But now, he said, Cantor introduces a very useful mathematical instrumentation which is implicitly a way to integrate infinite discontinuities. Using the already elaborated simple image in which the sides of an infinite polygon are represented by a a a a a. . . and the angles between the sides by \*\*\*\*. . . , then to represent the polygon as sides plus angles, we write: a\*a\*a\*. De Paoli then shows that a transfinite *number* means the following.

If W is the transfinite of a a a a a . . . , it can be thought of in all the following ways:

• It is a transformation of the *linear actions* of the sequence 1, 2, 3, 4 . . . a a a a a, in a quantum of action, a relatively completed action.

FIGURE 4 Georg Cantor's infinite-sided polygon



In Georg Cantor's representation, we consider an infinite-sided polygon, where a represents the sides and \* the angles. The infinite polygon is then written as a\*a\*a\*... Since this is definable, the whole infinite process can be called W, which is the infinite polygon. Then another set W+1, which is transfinite to W can be conceived.

• It is at the same time the *maximum* of a a a a a . . . and a *minimum*.

• It is transcendental to a a a a a . . . , but present as ordering the symmetries or metric of a a a a a . . . This means that it defines the changes as angles between a a a a a . . . , that is: a \* a \* a \* a \* . . . Thus W is the infinitely dense set of discontinuities of first species (that is, the angles) in the line.

But now W itself can be posed as object, and become a "Many." Technically this means that Cantor develops a transfinite arithmetic. We get the series W, (W+1), (W+2)..., etc. which now projects in A as:

$$(a*a*a*a...) \Delta (b*b*b*b...),$$

where  $(a \neq a \neq a) = W$ ;  $(b \neq b \neq b) = (W+1)$ , etc., and  $\Delta$ =the change between W and (W+1) or the change of assumption or postulate in the (a, b area), or *discontinuities of the second species*, as they are called.

Now, (W\*W1\*W2...) must reflect the same principle—they are causal evolution and so integrable, in the Cantorian sense, by their transfinite (let's call it N1).

Now, NI defines all the functions (a, W) and the disconti-

nuities  $(*, \Delta)$ , where:

a=theorems

W=the assumptions (postulates)

 $\star,\Delta$ =the discontinuities of first and second species (i.e changes in the theorems and postulates).

So, we can say:

• The transfinite is an integral but not as a linearization.

• The first complete operation of transfinite we obtain that is N which defines (a, W, \*,  $\Delta$ ) is the first relative continuum (or real curved action).

• It is a continuum which has infinite discontinuities in it, like the circle, which in reality, as real curved action, is not reducible to being linearly differentiable at any point contrary to the usual assumptions.

Cantor then restates all of this under his famous Three Principles of Construction. These are:

1) Principle A: Construct by adding; use order as more than or less than, etc., in this way producing finite aggregates, the discrete.

2) Principle B: Given an infinite series of type A, unlimited, I can conceive a new number, W, which is intended to be the expression of the fact that the totality of A is given in its lawful succession. Then W can be considered also as the limit of the series and transfinite to it. That is the generation of quanta of action.

3) Principle C: The Sufficient Reason, or density, or ordering. It is reflexive. Between a, b, or N, N+1, there is an ordering of the discontinuities and *a reason* for the necessity of the generation of the superior power.

The unity of these three principles, and only that, is the *content* of what is called the "continuum" either as a relative or, as we will see, as an absolute. So here we have Plato again, but now with the full mathematically and physically precise implications to it. This defines man and action in the universe both as forms of the continuum. Cantor explicitly made the parallel between human creative powers, physical nature, and the transfinite type of action (see De Paoli, *21st Century* Summer 1991). This was the actual aim of Plato, Cusa, and Leibniz, as it was of Cantor and Riemann, whom we have to re-study from the standpoint of Cantor to specify the geometrical physical content of his work.

#### **Riemann and the transfinite**

Riemann once expressed the following thought:

"With every simple act of thought, something substantial enters our soul. . . . It appears to us as a unity, but as far as it is the expression of space-time, it seems to contain an inner multiplicity. . . . All thinking is the formation of such *geistmass* [substantial unities]."

And further:

"... We have at each moment a completed set of concepts with which we comprehend nature. But if something happens which is unexpected according to the concepts... that is which is inconsistent with them, then we have either to supplement them *with a new theorem*, or to rework them so as to resolve the inconsistency . . . so that our knowledge becomes more and more complete and probes more and more beneath the surface of appearances.<sup>1</sup>"

When we apply Cantor's understanding to the work of Riemann and Eugenio Beltrami, we have the best possible insight for defining metrical geometry as an expression of different types of curvature. Here it will have to suffice to say that the (a a a a a) of Cantor represents the zero curvature (the linear plane); the  $(*, \Delta)$  the negative type of curvature (the caustics), and the (Ws) the positive curvature (i.e., the sphere as the plane plus the point at infinity). This can be derived rigorously from what we have now presented. But the real continuum generates all three types of metrics. We see here the role of negative curvature as singularity, also as "binding force"; but it cannot and should not be seen in itself as in the chaos theory, but as part of the continuum.

#### **Cantor's 'Universal Hypothesis'**

Now, to Cantor's Universal Hypothesis. Let's call the universe the totality of the transfinite actions and everything else existing:  $(N, N1, N2, ..., a b c d e. ..., ****..., \Delta\Delta\Delta ...)$ , that is, the [A+B+C] Principles of constructions. The usual philosophical and physical notion of the universe can then be thought of as follows:

1) It is discrete and unlimited.

In this case, it has no transfinite. But that instantly eliminates any type of causal changes between N, N+1, or even down to denying any changes of the type (a,b,c). Remember that causally ordered change for unlimited series is the same as transfinite existence. Thus, in this system, if any evolution happens, it is unintelligible—it is *chaos*.

This is the open universe of pure negative curvature.

2) It is discrete and limited (Bertrand Russell).

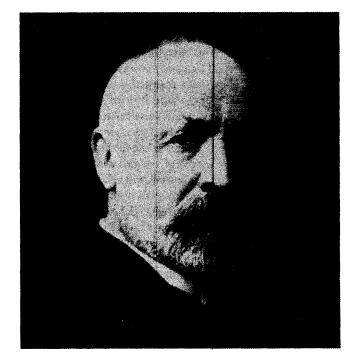
This, too, eliminates the transfinite. We are at Level A. We have pure linear action, a pure Euclidean universe. The universe in which a computer could determine present past and future.

Both 1) and 2) imply that there is no well-ordering, and any active principle including Life and creative Mind cannot have real existence and has to disappear and be ultimately reduced either to linear changes or to occult causes.

3. It is a Cantorian continuum.

But then to be well-ordered, closed, it is necessary that it is "brought into being" by *an absolute transfinite (infinite)*, transcendental to it, but also in it, immanent as *Logos*, as the ordering of it.

In this *curved* universe, creative minds and life can exist, as created, in the way we experience them at present. With this, among other things, Cantor established a new logical proof of the Necessity for and Sufficiency of the existence of God as Creator. But this also helped him to solve crucial issues of formal mathematics which lead to the issues which the 20th-century logician Kurt Gödel addressed later on. So



Georg Cantor

we have seen in a rapid-fire fashion, how the solution to the circle-squaring paradox, the assumption of curved action and curved space as primary over the visible linear one, can be represented by Cantorian transfinites. And this, then, defines the existence and action in physical space of what is usually considered the spiritual action of our mind as it makes creative changes.

This needs to be consistent with the assumption of the existence of God, the Creator, in the way that Christianity represents it. What was implicit in Plato becomes explicit in Cusa, Leibniz, and Cantor. Science is not only a relation between man and nature, but a specific type of such relation, where both man and nature can be viewed as created things which mirror the universe, but man, and man alone, can also act in the image of God the Creator. In Cantorian terms, man and living things are different transfinites, or order types, even if self-similar to the Absolute Transfinite to which we associate God. The self-similarity of the "continuum" in Cantor's work, expresses the same issue geometrically.

#### **Modern trends**

The inconsistencies inherent in the view of a discrete universe, measurable only as A is leading today more and more to the collapse of the Newtonian and LaPlacian matrix in physics—particularly of LaPlace's determinism, which he reduced to the famous statement "I do not need to introduce the hypothesis of a God," or that a sufficiently big mechanical device (computer) could know everything in the universe. Reality is imposing itself. To deny the existence of *creative*  acts, or real curvature, becomes less and less possible, even if the establishment controlling science tries to hide it or to mystify it as they do by promoting the so-called Chaos Theory, or Complex Theory. If humanity accepts malthusian policy directives, and allows the destruction of technological development, especially of the capability for space colonization, the debate in science around these fundamental issues, will become as it was in the Middle Ages, tainted with mysticism and scholasticism. If we can defeat the malthusian policies, the solution to the issues of the curvature of physical space-time, along the lines indicated, will usher in new physical discoveries and advanced forms of technologies.

This is why it is not simply a logical debate. It is a political fight to maintain the "scientific matrix" which we have called Socratic Christian. It may become clearer to you now if I paraphrase a recent article by Cardinal Joseph Ratzinger. Around the issue of intelligibility and truth, Ratzinger writes, we can establish the clear distinction between Socrates-Plato, who believed that man can know the truth in connection with the Absolute, and those (referring to the Sophists) who believed that man could create on his own, *and arbitrarily*, the criteria governing his life. And then he adds: "The fact then, that Socrates, a pagan, could become in a sense, *the prophet* of Jesus Christ . . . is based on such fundamental issues."<sup>2</sup>

"That, I guess should stimulate your mind a little bit," De Paoli concluded. "As I stated at the beginning: Someone's ideas were transfinite to what I have presented. It is essentially Lyndon LaRouche's creative and political action that has presently *integrated* much of the work of the past and created the possibility for their continuation."

#### Notes:

1. "Fragments of a Philosophical Content" in *The Collected Works of Bernhard Riemann*, New York: Dover, 1953.

2. Il Sabato, March 16, 1991.

### Appendix: On the notion of continuity and the infinitesimal

The following comments may make clearer to some readers the fundamental issue addressed in the speech, which was an attempt to present in a simplified version the mathematical problem of continuity.

The notion of "continuity," as the Leibnizian principle of continuity or the Cantorian axiom of continuity, derives historically from the work of Eudoxus, Archimedes, Cusa, and Kepler, arriving at the modern form of the debate on the question of infinitesimals in which Cantor participated. The issue is crucial, not only epistemologically, but also as a matter of great importance in mathematics and physics.

The question may be best illustrated by considering the distinction existing between a circle and a polygon, between a line and a curve, or, better, between linear and curved action. If we do not admit the principle of continuity, the so-called Archimedean axiom, as it is called today, then essentially we rule out the possibility of ever bridging this gap. We operate only in the algebraic realm, and for that reason, we are restricted to the premise that the angle of an inscribed polygon, however manysided it may be, will always be smaller than the circumscribed circle; and also, that the "linear side" of the polygon, no matter how many times we divide it, will never become "zero" or really curved; that is, we have a non-Archimedean geometry, as it is called.

The transcendental numbers, and the concept of the transfinite more generally, establish precisely the common denominator for bridging the gap between these "two natures." Cantor's attack on the notion of infinitesimals is correct in this sense. But we must be careful not to confuse the term "infinitesimals," as it is employed in the 19th and 20th century, deriving from Cauchy (or the way Veronese used it), with Leibniz's terminology. In the modern, Cauchy version, the manifold in which the infinitesimal operates is reduced such that the impossibility of reaching the limit is built in from the start: we have the polygons, and we have the circle, and never can the two be brought together. Nor is Euler's approach, which demands an infinitely divisible manifold, any alternative.

In reality the alternative is precisely what Cantor had proved: The only meaningful notion of infinitesimals is found in the transfinite numbers. Thus, differentiation and integration are one common operation, a mirror image of transfinite action-that is the mixton in Plato, or the second principle of construction in Cantor. Cantor declares explicitly that he is able to find a "common measure" for continuity and discontinuity (Cantor Werke, p. 152). It must be emphasized that the continuum is not an object, it can be meaningfully understood only as the unity of the three principles of constructions. That, and only that, can bridge the "two natures" (as Cusa would have put it). The continuum is self-similiar but of different types of ordinals. By definition it creates segmentations or quantization, and thus cannot be reduced to a dichotomy between simple continuity and simple discontinuity.

—Dino de Paoli