Economics becomes a science

Lyndon LaRouche's Riemannian economic model —part 2

Final completion of the computer model for Lyndon H. LaRouche's Riemannian economic model was announced jointly by the U.S. Labor Party and Fusion Energy Foundation in New York, April 25. The first phase of the computerized "LaRouche Model," featured in the following *Executive Intelligence Review* Special Report, was completed when a series of trial tests designed to test the predictive power of the model were successfully run through the computer. According to the scientists running the LaRouche Model project, the predictive power of the model is "virtually 100 percent," and its degree of accuracy depends almost exclusively on the accuracy of the inputted statistical data.

In the first major test of its capabilities, the "LaRouche Model" was given statistical data from the 1968 to 1973 period and was then asked to predict what would occur if a 400 percent increase in oil prices were superimposed. In response, the computer model was able to produce charts and diagrams describing the behavior of such key categories as rates of capital formation, rates of productivity increases, rate patterns in consumer-goods industries, capital-goods industries, etc., which are virtually identical with what in fact occurred in the U.S. economy during the 1974 to 1978 period.

The LaRouche model of economic prediction and analysis was developed on instructions from Lyndon H. LaRouche, Jr., presidential candidate for 1980 and chairman and founder of the U.S. Labor Party. The research and development for the model was conducted by a team of Labor Party and FEF physicists, mathematicians, and economists headed by Dr. Uwe Parpart, the Director of Research of the USLP, who is also research and development director for the Fusion Energy Foundation.

The computerized LaRouche Model is based on the fundamental theoretical breakthrough in economic science which LaRouche accomplished during the early 1950s, the special significance of which is illustrated by his autumn 1978 *The Theory of the European Monetary Fund.* Although LaRouche's work is otherwise a continuation of the work of Plethon, Colbert, Hamilton, the Careys, and List, as well as incorporating essential contributions of Karl Marx, LaRouche, beginning in 1952, succeeded in solving the crucial problems left unsolved by all his predecessors in this field: the problem of developing efficient deterministic models for the rate of economic growth of economies under the impetus of directed rates of technological progress. This was solved with aid of the conceptual approach to relativistic physics identified with Bernhard Riemann's habilitation thesis, an understanding of Riemannian physics achieved with aid of the crucial work of Georg Cantor. The result is a unification of the problems of applying science to economy with the fundamental principles and methods of Riemannian physics.

by Uwe Parpart and Steven Bardwell

All presently employed national and world economic models suffer from two major interrelated deficiencies already identified in Part I of this report and reflected both in the models and in the data bases:

1. No distinction is made between *productive* and *nonproductive* economic activity and realization of economic output, where by productive we mean effecting a useful material alteration of nature resulting in tangible wealth (for detailed categorization see below) Consequently no concept of economic *surplus* in the sense of economic product representing "free energy" for the expansion of the productive base of the economy through investment in added productively employed labor and capital goods exists, and the concept of economic growth itself becomes ambiguous, even maligned as a cause of inflation.

2. Inadequate or no account is given of qualitative changes in the technology base of the economy, even though in the long run, such changes and their action on the productivity of labor are the only source of noninflationary growth. Appreciation of this fact itself, of course, presupposes the productive/nonproductive distinction. Another reason frequently advanced for not including technology changes in the usual models is that continuous models cannot accommodate them. This is true. Therefore, the Riemannian model proposed here is specifically geared toward the occurrence of discontinuities in one or more of the model's parameters. In fact, it is in order to emphasize this feature that the model is called "Riemannian": The 19th century German mathematical physicist Bernhard Riemann was the first to propose that the analysis of global phase spatial relationships proceed from the standpoint of the determination of the "shape" (or geometry) of the phase space or manifold by means of the distribution of the singularities of the parameters spanning the space. In his discovery and description of the phenomenon of shock waves Riemann gave a specific example of the evolution of a physical manifold toward a point of discontinuity and the subsequent qualitative reordering of the manifold as a result of the propagation of the singularity retaining its integrity as a new type of physical entity. Technological change will be seen to have shock wave-character in this general sense for our economic model.

Conventional models

Three conventional-type economic models shall now briefly be examined to illustrate the two points of criticism just made and to show how the indicated defects lead to gross predictive and policy failures. In presenting this short critique we want it to be understood that we do not overlook the fact that precisely to the extent that some of the examined models have about the same predictive power as the charts of an astrologer, their main purpose may in fact be the same as that of such charlatans in the service of the King: to provide some semblance of objective justification for the credulous for policies pursued for entirely different reasons.

First, there is the so-called *naive* forecasting method which hardly deserves the name model. It proceeds by establishing certain historical trends for a given variable or set of variables and then more or less uncritically projects that same trend (positive or negative growth rate) into the near or distant future. Such simple inductive procedures are open to so many different and obvious objections that it is hardly worthwhile to bring our two points above to bear on them.

Significantly, however, economic models of the second category to be considered, the *econometric models* forecasting *national income account statistics* (such as the U.S. Department of Commerce Bureau of Economic Analysis [BEA] quarterly model), are subject to the same inductive fallacy—and in fact doubly so. They attempt to forecast the values of variables such as personal income, government spending, gross national product, etc. on the basis of some combination of

1. historical, or lagged values of the variable in question;
2. other variables related to it by a set of linear equations, and
3. exogenous variables determined by factors not covered by the model at hand.

The first of these forecasting methods, of course, is just our above naive extrapolation; but basically so is the second. The linear equations establishing the relationships between the different variables of the model normally reflect and are justified by the model builders through reference to historically observed *statistical correlations* between the variables rather than purporting to express actual *causal relations* in the economy comparable, say, to the laws of classical physics. Thus, the theory expressed by the model's equations has itself the epistemological status of a simple inductive generalization no different than naive extrapolation.

Most professional economists today will probably contend that economics admits of no other kind of "lawfulness" and theorizing, and that our criticism of the standard econometric models is therefore vacuous. We are not impressed by that point: Aside from their epistemological inadequacy, the forecasting performance of existing models is notoriously unreliable, while the causal analysis of the economy we are proposing will recommend itself mainly by its predictive accuracy. There is no reason to drive a wedge between physical and economic theory. In physics, when someone claims he knows that under normal conditions water boils at 100°C, because he has made a long series of boiling experiments all with similar outcome, we do not accept that as a valid reason. Instead, we ask for a more reliable explanation which must contain some reference to a cause/effect relationship between the heating of water and the onset of cavitation (boiling).

Similarly, in economics, a statement that historically a change in such and such variables has generally produced such and such overall growth rate, should be regarded as at best incomplete, leading to an answer of the more basic question of *why* the variables examined showed the observed behavior.

There actually exists in today's economy a crucial phenomenon which just begs for the proper kind of causal analysis—the phenomenon of inflation. The very fact that inflation has so stubbornly resisted the econometricians' best predictive efforts demonstrates the incompleteness of their models caused by an inadequate choice of parameters and relations between these parameters. Einstein never tired of making the same point regarding the quantum mechanical uncertainty relations; they must be taken as a signal of incompleteness *with respect to the chosen parameters* and phase space relationships, but not as a sign of the ultimate causal incompleteness of physics. We shall show in the following how it is precisely the distinction between the productive and nonproductive realization of the total economic product that allows for a choice of parameters or variables which span the kind of phase space in which causal determination of economic behavior is possible. This identification of the causal parameters then becomes the basis for national policy intervention into the economic process as well as of reliable forecasting.

A third type of model to be considered briefly is the Leontief-type input-output model. In outward appearance at least these very differentiated models mapping the flow of goods and services between the different sectors of the economy are concerned with establishing substantive producer-consumer relationships and not just with statistical correlations between selected variables.

Typically these input-output or interindustry models display in matrix form the percentages of the total product of a given industry (or sector of the economy) consumed by the totality of other industries (or sectors) or going to "final demand" (see Figure 1). Such matrices can then be used to estimate required industry-by-industry inputs to obtain desired outputs both in the overall and with respect to certain specific output categories.

There is no question that the data displayed in input-output matrices can play a valuable role in economic policymaking; however, when applied to the problem of determining or forecasting economic growth in any time-frame except the immediate short run, input-output analysis suffers from the same fundamental flaw as the econometric models. All different output categories of the total economic product are treated on a par with each other, and no distinction is made between their productive and non-productive consumption. However, it is only through the introduction of such a distinction that the future capacity for economic growth resulting from the consumption or realization of a certain type of product (mainly through the enhancement of the productivity of labor based on the rate of introduction and propagation of new technologies) can be judged.

Simple production-consumption relations will not do, and input-output tables ultimately leave obscure the causes of economic expansion and contraction.

Perhaps the most serious indicator of insufficiency of conventional economic models is found, however, in their assumption of a simple continuity in the economic variables of interest. The related assumptions of a fixed set of economic relations and the imposition of a continuity condition on those variables are fundamental to all the present types of models—econometrics demands a continuous manifold for the solution of its

Exchange of goods and services in the U.S. for 1947

INDUSTRY PURCHASING

INDUSTRY PRODUCING

		agriculture and fisheries	food and kindred products	textile mill products	apparel	lumber and wood products	furniture and fixtures	paper and allied products	printing and publis[illegible]
		1	2	3	4	5	6	7	
agriculture and fisheries	1	10.86	15.70	2.16	0.02	0.19		0.01	
food and kindred products	2	2.38	5.75	0.06	0.01	*	*	0.03	
textile mill products	3	0.06	*	1.30	3.88	*	0.29	0.04	
apparel	4	0.04	0.20		1.96		0.01	0.02	
lumber and wood products	5	0.15	0.10	0.02	*	1.09	0.39	0.27	
furniture and fixtures	6			0.01			0.01	0.01	
paper and allied products	7	*	0.52	0.08	0.02	*	0.02	2.60	1[illegible]
printing and publishing	8		0.04	*					0[illegible]
chemicals	9	0.83	1.48	0.80	0.14	0.03	0.06	0.18	0.[illegible]
products of petroleum and coal	10	0.46	0.06	0.03	*	0.07	*	0.06	[illegible]
[illegible]	11	[illegible]	[illegible]	0.01	0.02	0.01	0.01	0.01	

The above matrix shows, in part, the transactions of the U.S. economy during 1947 for which preliminary data were compiled by the Bureau of Labor Statistics. The matrix is taken from a larger one published in *Input-Output Economics* by Wassily Leontief (Oxford University Press, New York: 1966). Each number in the body of the table represents billions of 1947 dollars.

In the vertical column, the entire economy is broken down into sectors. The same breakdown is repeated in the horizontal row. When read horizontally, the numbers indicate shipments to other sectors. When read vertically, the numbers show what that sector consumes from other sectors. Asterisks stand for sums less than $5 million.

equations, and input-output analysis relies on the economy's being describable by the same size matrix over time.

It has been well known in physics since the time of Riemann and Boltzmann that these assumptions have drastic consequences for the system they describe—the assumptions of linearity (in the sense of fixed laws) and continuity, while both "local" statements about the system, have profound, global consequences. In the case of physics, these two assumptions lead to the Second Law of Thermodynamics (see Morris Levitt's article on this subject in the *Fusion Energy Foundation Newsletter*, September, 1976). This is a very deep result, first rigorously shown by Bolzmann's famous H-Theorem: a linear, continuous system is subject to an inexorable increase in entropy, the eventual running down and disintegration of order. The critical point for discussion is that the *same proof* holds for an economic model: the assumptions of linearity and continuity lead, by the same reasoning, to the necessary entropic consequence for an economic system unfortunate enough to satisfy these assumptions.

Obviously, however, neither history nor real economic systems satisfy either of these assumptions—economic change, especially technological change, happens discontinuously and, through this discontinuous process, the laws describing that economic development change qualitatively. The only presently existing mathematics sufficiently powerful to describe evolution of this sort is that outlined by Riemann and his school.

It is critical to note that the surprising indifference of models like the Department of Commerce model to growth rate—they predict equally healthy futures for the economy almost independent of growth rate—and alternatively, the gloomy necessity of zero growth deduced from system models like the "Limits to Growth" study of Meadows and Forrester, both stem from Boltzmann's H-Theorem. These models, to take two examples, had zero growth and the impossibility of technological progress built in from the beginning in their most fundamental assumptions about the mathematics relevant to economics. They "prove" the possibility or necessity of zero growth as a purely circular consequence of their axioms.

The BEA on its econometric model

Ever wonder how the government comes up with its economic forecasts of what Americans can expect in the way of inflation and other key trends? The Bureau of Economic Analysis offers this description of the accuracy of the econometric model they use to call the shots on the economy.

The whole inquiry—both its prediction and forecasting aspects—is aimed at the question: How reliable is the model as a forecasting instrument? The article does not provide an unambiguous answer to this question. However, both the quantitative error statistics and the analysis of turning point predictions show a substantial tendency toward deterioration as the prediction of forecast horizon lengthens. Since a large part of the impact of many kinds of government economic policy actions occurs several quarters after such actions, further improvements in econometric modeling are desirable

An econometric model is a set of equations comprised of behavioral relationships plus "identities," or definitional relationships. The behavioral relationships are specified (as far as possible) on the basis of economic theory and are estimated by fitting regressions to actual data. A basic assumption is that the relationships are "stochastic." That is, even if all of the important causal determinants are included as explanatory variables in an equation and the form of the equation is properly specified, there remains a random or unexplained error term (often called "disturbance") which represents the net effect of the myriad other forces that are acting on the dependent variable.

The principal categories of reproduction analysis

Since the mid-1950s economist Lyndon LaRouche has proposed a causal method of economic analysis which has served as the basis for the economic forecasting and planning model developed over the past several months by the writers and several collaborators. The latest popular presentation of LaRouche's method is contained in his October 1978 piece "The Theory of the European Monetary System." (New York: Campaigner Publications.)

As opposed to the economic models just reviewed, LaRouche's analysis does not attempt to aggregate a whole economy from its component parts expressed as variables and their interrelations, but proceeds from the economy as a whole as the primary datum. In analogy to a living organism the economy is viewed as a reproductive system by which a given population produces and reproduces the material conditions of its existence, and in which the principal quality to be measured is the *negentropic* contents (enhanced reproductive capacity) inherent (or lacking from) societal transformations represented by successive epochs of the production-consumption cycle.

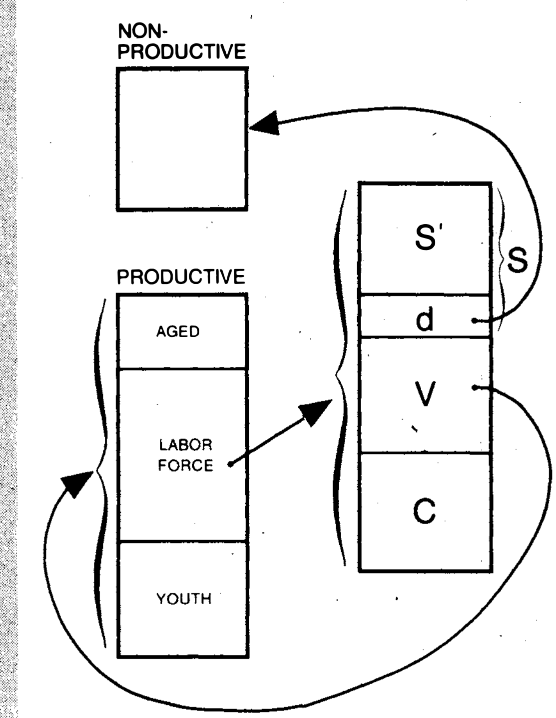
The reproductive categories employed by LaRouche (and displayed in their interrelations in Figure 2), as based on the division of households into productive (income derived from labor employed for the production of tangible wealth) and nonproductive, are as follows:

(1) v = *variable capital* = portion of the total product (output) produced through a given production-consumption cycle representing the cost of reproducing the productive labor force at the same cultural-material level;
(2) c = *constant capital* = portion of total product representing the cost of replacement of plant, equipment, and raw materials at current level and quality of production;
(3) s = *surplus product* = portion of total product exceeding the quantity c+v
(4) d = *nonproductive consumption* = portion of total product representing the cost of reproduction of non-productive labor in the private and public sectors at current level;
(5) s′=s−d = *absolute surplus* = portion of total product available for reinvestment for expansion of v, c, and d.

In the discussion of the data base of our model we will later explicitly relate these categories to categories familiar from Department of Commerce and similar government statistics.
For the time being we note that on the basis of the variables v, c, s, d, and s′, (aside from the time variable t, the only variables to be employed in our base model) certain ratios can be defined which represent important performance characteristics of the economy:

1. s′/(c+v), the rate of surplus production, is a "free energy" ratio, and represents the economy's capacity for expansion;
2. s/v, the rate of surplus at current reproductive costs of labor, is closely related to productivity as defined in government statistics and will be referred to as productivity in the following;
3. v/c is a measure of the capital or labor intensity of the economy, etc.

For our present discussion, the behavior over time of the ratio s′/(c+v) is the most significant quantitative indicator. Rising values of this ratio represent the desired negentropic quality of the economy as a whole *caused* by investment in advancing technological capacity and necessary concomitant investments in improved specific skill and overall cultural levels of the labor force.



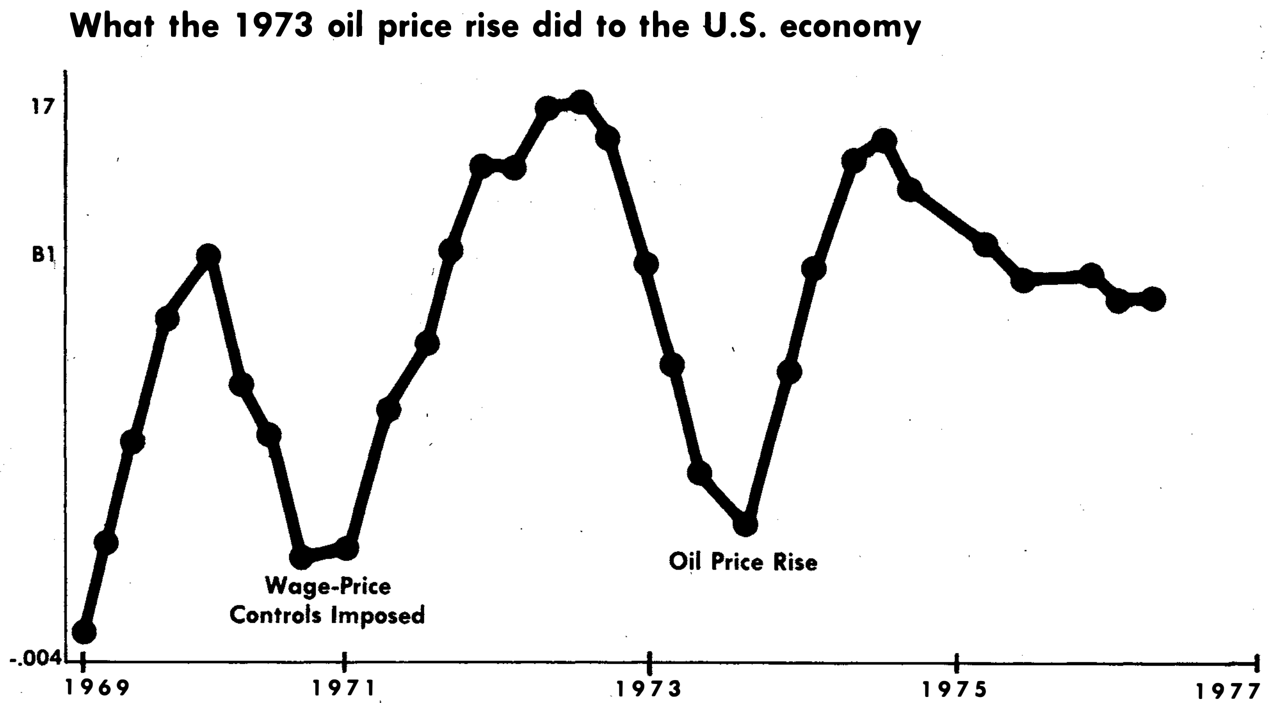
The Model[1]

The *base model* of an arbitrary economy to be described at this point—a more sophisticated version will be introduced below—consists of a set of three ordinary differential equations governing the time rate of change of the just introduced variables v, c, s′, and d. The equations relate these variables to three ratios of values of the variables assumed constant for the length of the reproductive cycle under consideration.

In essence, these values are politically determined, and reflect the results of public and private sector policy choices operative in the economy. The required ratios are:

α = the fraction of absolute surplus s′ reinvested in v;
γ = d/v=the ratio of non-productively invested surplus to v in the current cycle;
δ = (s′+d)/v=the ratio of surplus production to v (productivity) in the current cycle.

With these variables we can write down the following equations for v, c, and s=(s′+d):



The overall effects caused by the rise in the price of oil after 1973 can be seen in the dramatic changes in the ratio S'/(C+V). As S' or social surplus, which can be reinvested into plant and equipment (C) or the maintenance of the productive population (V) dropped, so did living standards, while the actual productive capacities of U.S. plant and equipment plummeted.

(1) $dv/dt = \alpha s'$

(2) $dc/dt = (1-\alpha)s'$

(3) $d(s'+d)/dt = d(\delta v)/dt = v(d\delta/dt) + \alpha\delta s'$

The first two equations simply express the fact that v and c change only as a result of the reinvestment of some surplus or profit (either positive or negative) in these categories. The third equation defines changes in surplus production as the sum of changes in productivity (s/v) and in the absolute size of v. It is this third equation, in particular, which expresses the essential causal relation in the model: surplus is created by the productive employment of labor; its size is proportional to v, with *productivity the crucial constant of proportionality.*

It is clear that, in most situations, using only these three equations would be a gross oversimplification of any economy. It is straightforward to write down the more complex sets of equations which are required for a multisector economy, each of whose internal dynamics is governed by similiar equations, but with different ratios. Below, we will show a simple example of such a system of coupled economic sectors.

An important capability that such a scheme of coupled sectors provides is the analysis of the interaction between the underdeveloped and developed sectors of the world economy. Without a doubt, the most striking feature of the world economy is its division into two sectors with grossly differing values for the ratios α and δ. The dynamics of the world economy depend on the interaction between these two subsystems.

Second, this multisector approach provides a natural scheme for examining the impact of the world economy on a subsumed (smaller) sector of that economy. Since the impact on a subsector of any pattern of world economic development will depend greatly on the level of industrialization, living standard, and productivity of that subsector, it is essential to have a way of measuring the effects of the world economy on any given subsector. The model provides a powerful way of studying the effect of different scenarios of world economic evolution on a single industry or state.

1. We employ the term "model" only with great reluctance. It indicates a very tenuous relationship between analysis and reality, the kind econometricians regard as necessary, but we find quite unacceptable. We claim for our "model" the same status normally claimed for physical theories.

Figure 4

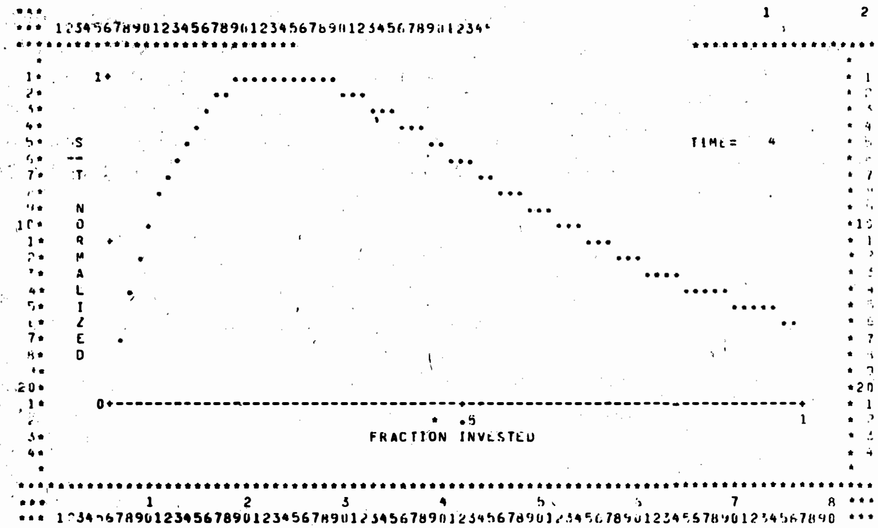


Figure 5

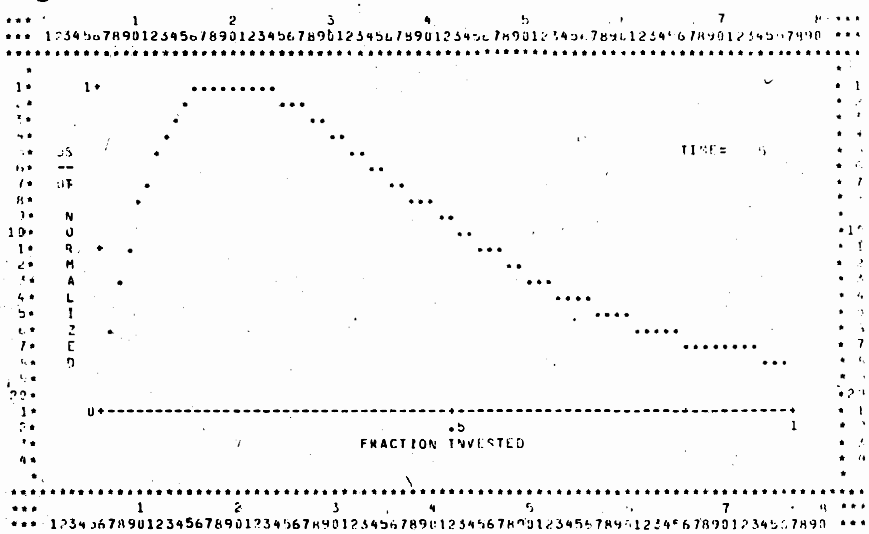
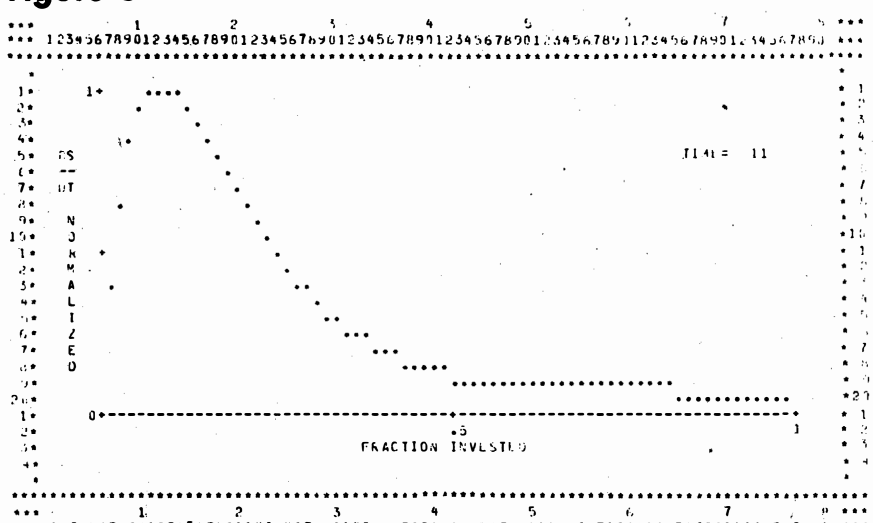


Figure 6



Semiquantitative results from the model

These three graphs were produced by test runs with a computer code for the Riemannian economic model. They examine the dependence of the growth rate of the underdeveloped sector as a function of the fraction of the advanced sector's surplus (profit) which is invested in the underd veloped sector. This dependence is shown for three periods as the underdeveloped sector grows. Most striking is the fact that there is an optimum fraction for most rapid growth in the underdeveloped sector—it is possible to invest too little and too much in the underdeveloped sector. The second possibility arises because the growth rate in the underdeveloped sector depends heavily on the growth rate in the developed sector (since a fraction of the underdeveloped sector's reinvestment comes from the advanced sector); hence, a fall in the developed sector's growth rate ultimately affects the underdeveloped as well. Also note that this optimum falls over time—that is, a successful development policy is possible.

In Figures 4, 5, and 6, the values of the abscissa (fraction invested) represent the percentage of advanced sector surplus invested, from 0 to 100 percent, in the developing sector over time (4, 5, and 11 production cycles in our examples), and the values of the ordinate (from 0 to 100 percent) represent the growth rate produced in the developing sector.

Important analytical solutions

Although these equations are clearly intractable in their general form with any methods besides numerical ones, there are certain special cases in which they can be solved analytically. These analytical solutions reveal some important characteristics of the model equations and give significant insights into the underlying dynamic and structure of the numerically derived solutions.

Single sector solution

In many cases, two important simplifications can be made to equation (3), reducing it to the form:

(3') $ds'/dt = \alpha(\delta-\gamma)s'$

This equation will be a suitable replacement for (3) whenever the rate of change of δ and γ are much less than the rate of change of v.

From this simplified form of the equations, several interesting conclusions can be derived. Most important is the dependence of s' on the various ratios. It is easy to integrate (3') for a general time dependence of α, γ, and δ, with the result:

$$s' = s_0'\exp[\int_0^t \alpha(\delta-\gamma)dt'].$$

The growth of s' is found to be highly dependent on the composition of reinvested surplus, α—it is possible to some extent to trade off γ and δ, letting one decrease if the other does so, and still maintain the rate of growth. However, the influence of α is essentially different. Any small change in the composition of reinvested capital greatly affects the rate of change of surplus. This is an important qualitative result, since it demonstrates in both a positive and negative sense the importance of the cost of replacement of c. A small cheapening (for example, a new mining technology) will greatly increase the growth of profit. The introduction of any method of cheapening c is disproportionately advantageous for this reason. By the same token, a small increase in the cost of c can dramatically decrease the rate of growth. This was the case, for example, with the rapid increase in the price of oil over the last five years.

We have just completed a case study of this situation and found that our qualitative prediction based on the analysis of the just exhibited analytical solution to the model equations is fully borne out by the quantitative results (see Figure 3). A more general point is this: the now quite common scare stories about "running out of everything"—oil, precious metals, timber, etc.—even if they were true, are relevant to economic growth only indirectly. What matters is not the absolute availability of raw materials, but their availability *at a given price*. Thus it will not do to make provisions for the replacement of certain energy sources, for example, only when we are already at the borderline of marginal exploitation of the source. New sources must be brought on line early enough so as to be phased in well before depletion costs of current sources rise to a point where the entire economy suffers and is reduced in its capability of developing new energy technologies at the required rate. From these considerations the fallacy of applauding higher energy prices (whatever the cause for such price increases) because they make the development of new sources profitable ought to be obvious.

Two-sector solution

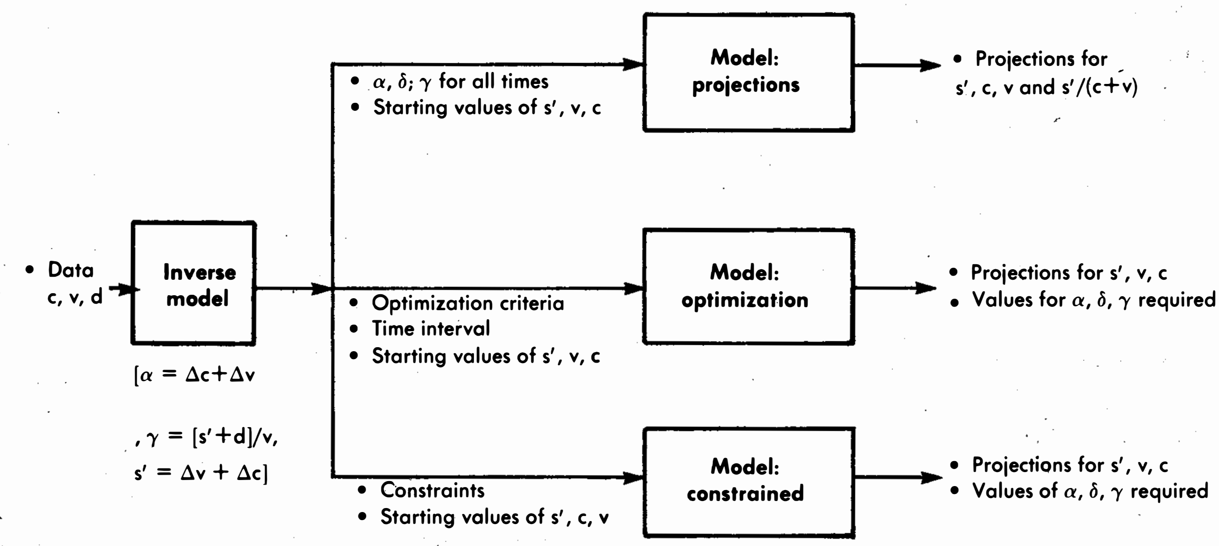
Under the assumption of constant (historical) values for the ratios, an interesting solution to the case of a two sector world economic model can be obtained. A realistic first approximation to the relation between the advanced and underdeveloped sectors, (sector 1 and 2 respectively), is the investment of a fraction, ϵ, of surplus from the advanced sector into the underdeveloped sector. Under these assumptions, the coupling between the two sectors is only through the fraction ϵ and the equations take the form:

$$ds_1'/dt = \alpha_1(\delta_1-\gamma_1)s_1'(1-\epsilon),$$
$$dv_1/dt = \alpha_1 s_1'(1-\epsilon),$$
$$dc_1/dt=(1-\alpha_1)s_1'(1-\epsilon)$$
$$ds_2'/dt = \alpha_2(\delta_2-\gamma_2)(s_2'+\epsilon s_1'),$$
$$dv_2/dt = \alpha_2(s_2'+\epsilon s_1')$$
$$dc_2/dt = (1-\alpha_1)(s_2'+\epsilon s_1')$$

It is straightforward to derive from this an algebraic expression for the growth rate of the underdeveloped sector as a function of ϵ. In Figure 4 this growth rate (normalized) is plotted as a function of the fraction of surplus invested from the advanced sector. There are two remarkable things shown by this very idealized model; first, the growth rate has a maximum. That is, there is an optimum fraction of surplus to be invested—it is possible to invest too much in the underdeveloped sector. The consequent decrease in the underdeveloped sector's growth rate comes from the fact that a depletion of the advanced sector's capability to grow will adversely affect the growth of the underdeveloped sector as well. However, it is striking that there is an optimum for this fraction.

Secondly, as Figures 5 and 6 show, this optimum decreases with time. That is, as the underdeveloped sector grows, the amount of advanced sector surplus reinvested for maximum growth decreases, as it should in any successful development effort.

Figure 7
The Riemannian economic model



General utility of the base model

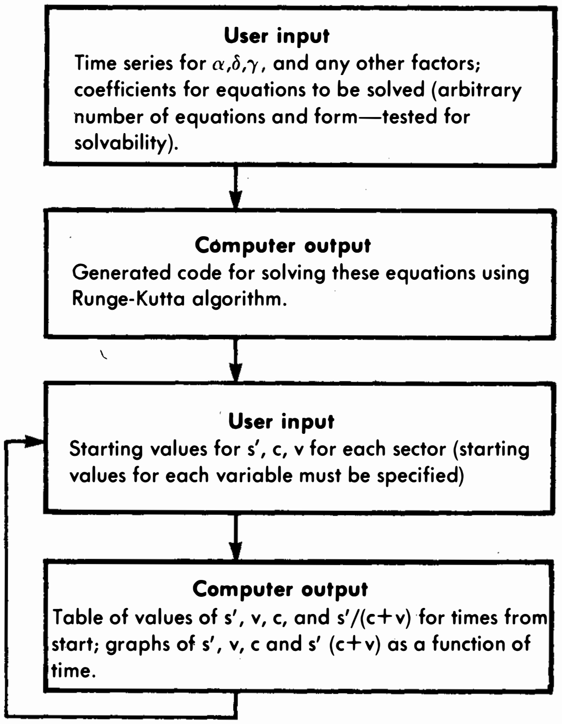
We have so far examined the use of our base model in two specialized areas of application: in what might be called "impact" studies evaluating the effect of significant short-term fluctuations of one variable while others are held relatively constant, and in two-sector studies focusing on the optimal allocation of surplus product. The general range of the model, however, is much broader and is indicated in the first flow chart (Figure 7). (The second chart [Figure 8] merely provides further details on model use and construction.)

The principal intended forecasting use of the model (presently limited to the U.S. economy, but soon to be extended to other advanced sector economies pending preparation of data base) proceeds along the following path (cf. Figure 7): we start by inputting a time series of values for v, c, and d and on that basis derive a set of historical values for the ratios of α, γ, δ and for s'. We can then reset these values at different levels reflecting arrays of possible private and public sector policy decisions affecting the ratios, and compute the outcomes for economic growth. As indicated above, α reflects a variety of investment decisions with respect to v, c, and d, but is being watched primarily for the impact of raw materials pricing; President Carter's recent oil price decontrol decision is a case in point; results of an impact study now in progress will be published in this magazine shortly.

Ratio γ principally reflects the impact of government spending in such areas as transfer payments and defense spending. Ratio δ is generally, but not usually immediately, affected by investments in new technologies, government investment credit decisions for such purposes, etc. In the short term the ratio reflects capacity utilization and other familiar factors impacting productivity. Input of different sequences of values for our ratios now yields different output sequences for v, c, s', and s'/(c+v).

We will normally publish not just the one sequence we judge the most likely, but several competing ones, in order to make transparent the impact of economic policy decisions, and in order, if you will, to tell people who to blame for outcomes deemed undesirable. Our principal published indicator will be a value for "real GNP" equal to the predicted value of the sum v+c+s.

Figure 8
Detail of Model: projections



The difference between Department of Commerce GNP and "real GNP," signifying the difference between total unevaluated economic growth and productively realizable growth, will for the first time allow the drawing of accurate conclusions about the expected rate of inflation for constant employment figures.

As shown in the flow chart in Figure 7, our model is intended, and will also be used, for economic planning purposes. Indeed, it is the principal merit of a causal model that switching from predictive to planning uses requires no changes in the model base. Optimization uses will allow for determining optimal growth rates on the basis of specified initial values, while "constrained" model use allows, for example, the presetting of a certain growth rate and the evaluation of input requirements for the principa

The data base

In first approximation the preparation of the data base for our model poses relatively few difficulties; it involves a straightforward, essentially "algebraic," transformation of the U.S. Department of Commerce *Survey of Current Business* statistical categories into v, c, and d as follows (page references are to the "current business statistics" section of the *Survey*):

Variable capital v: defined to be the portion of sales used for replacement of manufacturing and industry labor force (nondefense).

v=(food, housing + energy) · ratio of goods producing workers to total workers · ratio of average manufacturing wages to average of all wages =TLFC · ratio 1 · ratio 2 (TFLC = total labor force consumption)

=TLFC · ratio 1 · ratio 2

where food, housing = HGA + CS + AE − NPAE + PRC + PHR, where HGA = home goods and apparel (S-6); CS = consumer staples (S-6); AE = automotive equipment (S-6); NPAE = nonpassenger car automotive equipment (= TB · AE / PC, where TB = trucks and buses factory sales [S-40], PC = passenger cars, factory sales [S-40]); PRC = private residential construction (S-6); PHR = public housing and redevelopment (S-6).

where energy = EPSR · REC / TEC + GSRC, where EPSR = electric power sales revenue (S-26); REC = residential electrical consumption (mnkwhrs); TEC = total electrical consumption (mnkwhrs); GSRC = gas sales to residential customers (S-26).

where ratio 1 = GPW / TENA, where GPW = goods producing workers (1000s) (S-14); TENA = total employees, nonagricultural (1000s) (S-14)

where ratio 2 = AWEM / AWEP, where AWEM = average weekly earnings, manufacturing employees (S-16); AWEP = average weekly earnings, all private employees (S-16).

This gives the final formula:

v = (HGA + CS + AE · (1 − TB / PC) + PRC + PHR + EPSR · REC / TEC + GSRC) · GPW · AWEM / (TENA · AWEP).

Constant capital c: defined to be the portion of sales used for replacement of plant, equipment, and raw materials at current level and quality of production. (All quantities in millions of dollars unless otherwise noted).

c = capital expenditures−transportation + energy where capital expenditures = MAS + CMAS + EDP − DCG − PAE + PNRCI + PCI, where MAS = materials and supplies (S-6); CMAS construction materials and supplies (S-6); EDP = equipment and defense production excluding auto (S-6); DCG = defense capital goods; PAE = pollution abatement expenditures (S-6); PNRCI = private non-residential construction (S-10); PCI = public construction, industrial (S-10).

where transportation = TTE − AE, where TTE = total transportation equipment (S-6); AE = automotive equipment (S-6).

where energy = GSI + EPSR · FEI, where GSI = gas sales to industrial customers (S-26); EPSR = electric power sales revenue (S-26); FEI = fraction of electrical power to industry = EI / TEC (EI = estimated industrial energy consumption = [GSIC/(GSCC+GSIC)] · ECI, where GSIC = gas sales to industrial customers [s-26]; GSCC = gas sales to commercial customers [S-26]; ECI = electrical consumption by commercial and industrial customers [S-26] [mnkwhrs]); TEC = total electrical consumption (mnkwhrs).

This results in the following formula:

c = MAS + CMAS + EDP − DCG − PAE + PNRCI + PCI − TTE + AE + GSI + (EPSR · GSIC · ECI) / (TEC · (GSCC+GSIC))

Nonproductive expenditures d: defined to be the portion of sales not used for replacement of manufacturing and industry labor force (nondefense) or for replacement of plant, equipment, and raw materials at current level and quality of production.

d = consumer goods, energy, transportation, and construction not used productively

where consumer goods = TLFC − v (see variable capital for description).

where energy = EPSR + GSR − GSI − EPSR · FEI − EPSR · REC / TEC − GSRC, where EPSR = electric power sales revenue (S-26); GSR = gas sales revenue; GSI = gas sales to industry (S-26); FEI = fraction of electrical output to industry (from c); REC = residential electrical consumption (mnkwhrs); TEC = total electrical consumption (mnkwhrs); GSRC = gas sales to residential customers (S-26).

where transportation and capital goods = DCG + PAE + NPAE + NATE, where DCG = defense capital goods (S-6); PAE = pollution abatement equipment (S-6); NPAE = nonpassenger automotive equipment (from v); NATE = nonauto transport equipment (S-6).

where construction = TNC − PRC − PNRCI − PHR − PCI, where TNC = total new construction (S-10); PNRCI = private nonresidential industrial construction (S-10); PHR = public housing and redevelopment (S-10); PCI = public construction, industrial (S-10).

This gives the final formula:

d = TLFC − v + EPSR + GSR − GSI − EPSR · (FEI + REC / TEC) − GSRC + DCG + PAE + NPAE + NATE + TNC − PRC − PNRCI − PHR − PCI.

This account also provides the reader familiar with Department of Commerce statistical categories with an explicit definition (at least in first-order approximation) of the categories v, c, and d and by implication of the productive/nonproductive distinction.

For forecasting purposes our data base so defined is more or less sufficient since we are not principally interested in the absolute values of our variables. In the case of planning uses of the model, this no longer holds true. If one wants to plan the economy of an underdeveloped sector country, for example, one must have reference to absolute values providing reliable information on the infrastructure, capital base, standard of living, productivity of labor, etc. in the economy. This necessitates much more far-reaching data transformations and *corrections* than indicated above. We are now in the process of developing such a data base for the economy of India; results will be published shortly in preparation for an international conference on the economic development of the Indian subcontinent.

Elaboration of the model

As successful and informative as the model in the above formulation remains, there is a fundamental feature of economic development which has remained an exogenous factor, namely, the interrelation between technological change, productivity, and growth of the economy. In the base model above, this relation must be supplied from the "outside" in the form of an empirically determined relation among the ratios α, δ, and γ.

To formulate a resolution of this insufficiency, some deeper introduction to the methodology of Riemannian mathematics is required. As noted above, the questions of the impact of technological change are geometric ones, not ones of a parameterization. That is, technological change introduces fundamental singularities into the actual economic process and precipitates, at these singularities, qualitative changes in the laws governing that economic development.

It is essential to realize that this jump-like behavior cannot be avoided—any model which assumes, explicitly or otherwise, that economic variables must be continuous, will not only fail to reproduce long-term economic behavior, but, more importantly, carries with it a set of axiomatic assumptions of the impossibility of technological change or realized scientific development. The historical fact is that such changes have taken place in a discontinuous manner, and, as we have seen, the assumption of continuity is equivalent to the assumption of a fixed mode of economic reproduction. Only by taking account of these discontinuous, qualitative changes can we deal with the central facts of human economic reproduction—namely, technological progress and increasing productivity through increasing cultural levels.

We have called the general approach of the model "Riemannian" because it uses the same mathematical tools that have been uniquely successful in Riemann's treatment of similar singular and nonlinear problems in physics. The crux of Riemann's method is his identification of the singularities in a process as the source of dynamics and internally determined geometry. Several points must be noted for further discussion[2]:

1. The set of variables required to specify the "state" of the economic system being modeled forms what Riemann called a "manifold." This manifold is a multidimensional space whose properties are determined by the form of the differential equations which describe the economic system. That is, the space is not specifiable beforehand—its metric, "flows," and the like are a product of the *imposition* of the time differential equations describing the evolution of the modeled economy.

2. The equations specify a set of trajectories through the manifold. The solution to the equations defines, in effect, the geodesics for the manifold and hence the actual trajectory used by the system.

3. However, as Riemann was at pains to point out, the interesting information for any manifold of this sort is given by the singularities which it contains. As is the case in complex physical systems,[3] these geodesics end or begin at singularities. At some point along the trajectory describing (instantaneously) a successful economy, some derivative will become infinite, or some ratio will have a zero denominator. This singular point represents the onset of a new mode of "interaction"—the necessity for a qualitative change in the economy, (the development of a new technology, an energy crisis, or the like).

4. The system of equations at this singular point cannot (usually) be made smooth again with any small change in parameters or any "adiabatic" change in the equations. The trajectories locally all share this singular point. A discontinuous change in the parameters or a qualitative change in the equations is necessary for the continued description of the system.

Figure 9
Aggregated economic data for the U.S. economy

Year	c	v	d	's'	α	δ	γ
1969	3.640	.810	3.470	.020	.499	4.259	4.2839
1970	3.630	.800	3.900	.639	.046	5.675	4.8750
1971	4.240	.830	4.270	.170	.705	5.349	5.1445
1972	4.290	.950	4.800	.800	.162	5.894	5.0526
1973	4.960	1.080	4.980	1.100	.036	5.629	4.6111
1974	6.020	1.120	5.370	.349	−.228	5.107	4.7946
1975	6.450	1.040	5.940	1.190	.277	6.855	5.7115
1976	7.310	1.370	6.960	1.010	.059	5.817	5.0802

The parallel to Riemann's treatment of the formation of shockwaves (discontinuous fronts of pressure in a gas) out of normal acoustic waves is instructive.

In his 1859 paper, Riemann described a physical process in which the propagation of a wave changed the medium in which the wave propagated, in such a way that the higher amplitude parts of the wave traveled faster than the low amplitude parts. By virtue of this process, the peaks of the waves catch up with the trough in a finite amount of time, and the wave "breaks" mathematically, leading to singular derivatives for the amplitude as a function of position. Riemann's contribution was the recognition that this singular point was not a mathematical fiction—as most mathematicians had assumed based on the assumption that the physical variables had to be continuous!—but rather, represented a qualitatively new feature in the system, a shock wave.[4]

Economic systems exhibit shock wave-like singularities at precisely the points of most rapid technological change, or the points where such change is necessary. We can replicate this behavior mathematically by an argument very similar to Riemann's. The simple form of the economic model described above permits propagation of an economy in one "direction" only, namely, the time direction. However, it is clear that economic evolution occurs in two almost independent directions—in time and in "technological development." Both of these "coordinates" are axes through which an economy can change, and, to a first approximation, economic evolution can occur in either "direction" almost independently. That is, an economy can exist by continuing in the same mode, at least for a length of time, without changing technology, and, in addition, a set of very rapid technological changes can occur in the space of only a few years, which could only be replicated within a given level of technology by many years of simple progress in the time "direction" alone.

Given this understanding of the geometric nature of technological development, we write, by analogy with a hydrodynamic treatment of the previously simple propagation time given by d/dt, the propagator:

$\partial/\partial t + u \cdot \partial/\partial x$.

That is, we allow differential (now partial differential) changes in two directions.

To give this so-called "convective derivative" meaning, we must specify the significance of x and u. The quantity x is relatively straightforward—it is clearly the level of scientific development of the economic system or sector under consideration. This is the "other direction" in which economic evolution can occur. Now, u must be the velocity or rate at which these scientific developments are translated into actual economic reproduction. Again, it is essential that these two facets of the propagation of an economy be distinguished; x is the distance covered by the economy in "scientific development" but this can only occur as the result of realization of such development in productive technology.

We now propose to recast our model equations (shown for a single sector) in the form:

$$[\partial/\partial t + u_1(\partial/\partial x_1)]s' = \alpha(\delta - \gamma)s' + (\delta - \gamma)v$$

$$[\partial/\partial t + u_2(\partial/\partial x_2)]v = \alpha s'$$

$$[\partial/\partial t + u_3(\partial/\partial x_3)]c = (1 - \alpha)c$$

where the u_i may or may not be the same. These equations have the striking property of supporting shock wave solutions! It is easily seen by looking at the first equation in the approximation that v is changing more rapidly than $(\delta - \gamma)$, in which case the first equation becomes:

$$[\partial/\partial t + u(\partial/\partial x')]s' = \alpha(\delta - \gamma)s'$$

Now, notice the obvious historical fact that as s' increases, so does u. That is to say, the larger the rate of surplus production, the larger the rate at which new scientific developments are assimilated. That means, mathematically, as Riemann showed for the very similar equations governing shock waves, that the troughs of the wave are overrun by the crests and a shock wave singularity develops in a finite time. In our equation, this singularity is reflecting the breakdown of the form of the equations themselves, just as it did in the case of shock waves. The formation of the shock wave represented the coalescence of the energy of the system in a new form, a coalescence which qualitatively changed the laws governing the future evolution of the medium of propagation.

In our Riemannian economic analysis, this velocity u is a close representation of the negentropic tendency of an economic system. Its realized tendency for technological change is quantified in u—not with the result that there is rapid expansion of the economy (this would be the result in a continuous system), but rather that it is a measure of the rapidity of onset of the shock wave-like singularity. Negentropy is not merely the tendency for internal elaboration and development, but much more essentially, the feature of the system which "forces" it to outgrow the present, fixed form of development.

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2. See F. Klein, Riemann's *Theory of Algebraic Functions*; U. Parpart, "The Concept of the Transfinite," *Campaigner*, Vol. IX, No. 1-2, Jan.-Feb. 1976.

3. S. Bardwell, "Solving the Three-Body Problem," *Fusion*, Vol. I, No. 8, June 1978.

4. For a detailed discussion of Riemann's treatment of shock waves see U. Parpart, "Riemann Declassified—His Method and Program for The Natural Sciences," *Fusion*, Vol. 2, No. 6, March-April 1979.